Synthesis of unknown Inputs PI and PMI Observers for Takagi-Sugeno augmented Models applied on a Manipulator Arm

Abdelghani DJEDDI[1], Youcef SOUFI[1], Salah CHENIKHER[1], Abdelaziz AOUICHE[1]

[1]Department of Electrical Engineering, Faculty of sciences and Technology, University Larbi Tebessi, Algeria

Abstract

This paper presents two approaches for simultaneously the state and unknown input estimation of a nonlinear system described by Takagi-Sugeno (TS) augmented models based on the unknown input observer under unknown inputs and outputs. The state and the inputs are estimated simultaneously using unknown inputs proportional integral (UI-PI) observers and unknown inputs proportional multiple integral (UI-PMI) observers. The sufficient conditions for the augmented TS models and each proposed observer are formulated by the Linear Matrix Inequality (LMI). An application based on manipulator arm actuated by a direct current (DC) motor is proposed to illustrate a comparison between the two unknown input observers.

Keywords: unknown inputs observer, proportional integral observer, multiple proportional integral observer, Takagi-Sugeno models, linear matrix inequality (LMI), augmented systems

Received: 28 January 2020

To cite this article:
DJEDDI A., SOUFI Y., CHENIKHER S., AOUICHE A., "Synthesis of unknown Inputs PI and PMI Observers for Takagi Sugeno augmented Models applied on a Manipulator Arm", in *Electrotehnica, Electronica, Automatica (EEA)*, 2020, vol. 68, no. 1, pp. 89-97, ISSN 1582-5175.

1. Introduction

Over the last decades, a various significant research activity has been focused on observer thesis for linear and nonlinear systems [1-7].

This thematic plays an important role in systems theory and control engineering. Generally, observers provide states estimation of outputs and unknown inputs [5], [8-12].

The state estimation of nonlinear systems represented by Takagi-Sugeno (TS) models has received a considerable attention and attracts a lot of researchers in the last decades [5],[13-15].

TS structure advantages are purported due to its simplicity, originates from the interpolation between linear systems [5],[16-17].

Thus, analysis and design methods developed for linear systems can be extended to nonlinear systems.

The stability problem analyses of TS systems and observer synthesis have been studied in many works [18-20] and the synthesis of TS systems observers for both cases when the premise variables are measurable (MPV) [5],[21], or when they are unmeasurable (UPV) [22]-[24], have been proposed.

In the context of state and unknown input estimation for nonlinear systems described by TS models, [9] has proposed an approach for the synthesis of PI observer for decoupled Multiple Model, [25] has proposed UI-PI observer with measurable premise variables (MPV) for singular systems, and [26] has proposed UI-PI observer with measurable premise variables (MPV).

However, PI observers can be used only if the unknown inputs are constant over the time, or when the variation of the unknown inputs are slow in respect to the dynamic of the system.

To overcome this problem, proportional multiple integrals (PMI) observers have been proposed to estimate all unknown inputs derivatives. PMI observers with MPV was proposed to estimate a large class of signals described by polynomial form for descriptor systems [27], PMI for Thau-Luenberger observer with UPV [23], and an unknown input PI observer with measurable premise variables [28].

The objective of this article is to design a new PI and a new PMI observer based unknown input observers for a nonlinear system described by TS models.

For this purpose, the proposed observers are used to estimate both the state and unknown inputs. The convergence conditions are derived in the linear matrix inequalities form.

The paper is organized as follows:

- section 1, is dedicated to the introduction;
- section 2 presents the considered class of nonlinear systems and the problem of the unknown input estimation;
- in section 3, the design of UI-PMI observers is synthetized;
- section 4 illustrates the obtained results using a model of manipulator arm actuated by a DC motor taking account the performances of the two proposed observers.
- finally, a conclusion and perspectives are given in Section 5.

2. Problem statement

The simultaneously estimation of states and unknown inputs for nonlinear systems represented by Takagi-Sugeno models can be solved by using the unknown input proportional integral observers.

In case when the unknown inputs are constant or have a slow variation, the UI-PI observers is sufficient, but in the general case, for a better simultaneous estimation of state and unknown inputs, the use of the UI-PMI observers gives more accurate results.

As mentioned above, the synthesis problem of UI-PI and UI-PMI observers when the premise variables are measurable is solved. However, the synthesis of UI-PI and UI-PMI observer premise variables are not measurable is developed and proposed in this work.

In this section, the unknown inputs are assumed to be constant, and the following assumptions and hypothesis are often not restrictive:

A1. The system is stable.

A2. The signals $u(t)$, $q(t)$ are bounded.

A3. $\dot{q}(t)=0$.

A. Takagi-Sugeno system

Consider the TS systems having measurable premise variables, with influence of unknown inputs:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{M} h_i(\xi(t))[A_i x(t)+B_i u(t)+E_i q_1(t)] \\ y(t)=Cx(t)+Eq_2(t) \end{cases} \quad (1)$$

The system can be written as:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{M} h_i(\xi(t))[A_i x(t)+B_i u(t)+E_i q_1(t)] \\ y(t)=\begin{bmatrix} C^{p,1} \\ \hline C^{p,2} \end{bmatrix} x(t)+\begin{bmatrix} 0 \\ \hline E^{p,1} \end{bmatrix} q_2(t) \end{cases} \quad (2)$$

where:

$C^{p,1} \in R^{(p-q)\times n}$ and $C^{p,2} \in R^{q\times n}$. $q_1(t)$ and $q_2(t)$ are unknown inputs.

The weighting functions $h_i(\xi(t))$ must satisfy the convexity conditions given by:

$$\sum_{i=1}^{r} h_i(\xi(t))=1 \quad \text{and} \quad 0\le h_i(\xi(t))\le 1 \quad (3)$$

Consider the new state g_f that is filtered version of $y(t)$ satisfying:

$$\dot{g}_f(t)=\sum_{i=1}^{M} h_i(x(t))[-A_f g_f(t)+A_f y(t)] \quad (4)$$

where A_f is a p dimensional stable matrix.

The augmented system is given as follows:

$$\begin{cases} \dot{x}_a(t)=\sum_{i=1}^{M} h_i(\xi(t))[A_{ai} x_a(t)+B_{ai} u(t)+E_{ai} q_1(t)] \\ y(t)=C_a x_a(t) \end{cases} \quad (5)$$

with:

$$x_a(t)=\begin{bmatrix} x(t) \\ g_f(t) \end{bmatrix},$$

$$q(t)=\begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix},$$

$$A_{ai}=\begin{bmatrix} A_i & 0 \\ A_{fi}C^{p,2} & -A_{fi} \end{bmatrix},$$

$$B_{ai}=\begin{bmatrix} B_i \\ 0 \end{bmatrix},$$

$$C_a=\begin{bmatrix} C^{p,1} & 0 \end{bmatrix},$$

$$E_{ai}=\begin{bmatrix} E_i & 0 \\ 0 & A_{fi}E^{q,1} \end{bmatrix}$$

The assumption (A3) is conventionally used for the theoretical demonstration of the convergence of the UI-PI observer, although in practice, we will find that we can overcome this, by increasing the gains of the observer in order to widen his bandwidth allowing thus taking into account the neglected dynamics.

However, this causes an increase in sensitivity to noise. The choice of the observer's gains is then determined by the satisfaction of a compromise between the robustness and the observer's performance [23].

B. Unknown input PI Observer Synthesis

The unknown input PI observer is given by:

$$\begin{cases} \dot{z}(t)=\sum_{i=1}^{M} h_i(x(t))[N_i z(t)+G_i u(t)+F_i\hat{q}(t)+L_{Pi}y(t)] \\ \hat{x}_a(t)=z(t)+Hy(t) \\ \dot{\hat{q}}(t)=-\sum_{i=1}^{M} h_i(x(t))L_{Ii}[y(t)-\hat{y}(t)] \\ \hat{y}(t)=C_a\hat{x}_a(t) \end{cases} \quad (6)$$

$N_i(t)\in R^{n\times n}$, $G_i(t)\in R^{n\times m}$, $L_i(t)\in R^{n\times p}$ is the gain of the i^{th} local observer, and H is a transformation matrix.

L_{Pi} represents the proportional gains and s the proportional gain and L_{Ii} the integral gains of L_{Ii} the integral gains of the observer.

$\tilde{q}(t)$ denotes the estimation error of the unknown inputs defined by :

$$\tilde{q}(t)=q(t)-\hat{q}(t) \quad (7)$$

The state estimation error is given by:

$$e(t) = x_a(t) - \hat{x}_a(t) \\ = (I - HC_a)x_a(t) - z(t) \quad (8)$$

and its dynamic can be expressed as:

$$\dot{e} = \sum_{i=1}^{M} h_i(x)[P(A_{ai}x_a + B_{ai}u + E_{ai}q) \\ - N_i z - G_i u - F_i \hat{q} - L_{Pi} y] \quad (9)$$

with $P = I - HC_a$

The expression (9) can be written as:

$$\dot{e}(t) = \sum_{i=1}^{M} h_i(x)(N_i e + (PA_{ai} - N_i - K_i C_a)x \\ + (PB_{ai} - G_i)u + (PE_{ai} - F_i)q + F_i \tilde{q}) \quad (10)$$

with

$K_i = L_{Pi} - N_i H$.

If the following conditions are fulfilled:

$$\begin{cases} P = I - HC_a \\ N_i = PA_{ai} - K_i C_a \\ L_{Pi} = K_i + N_i H \\ G_i = PB_{ai} \\ F_i = PE_{ai} \end{cases} \quad (11)$$

Then the state estimation error tends asymptotically towards zero and if the following conditions hold then, the equation (10) is reduced to:

$$\dot{e}(t) = \sum_{i=1}^{M} h_i(x(t))[N_i e(t) + F_i \tilde{q}(t)] \quad (12)$$

The augmented dynamic system of the state estimation error and the unknown inputs can be written as follows:

$$\dot{\tilde{q}}(t) = -\sum_{i=1}^{M} h_i(x(t))L_{Ii}C_a e(t) \quad (13)$$

The augmented dynamic system of the state estimation error and the unknown inputs can be written as follows:

$$\dot{e}_a(t) = \sum_{i=1}^{M} h_i(x(t)) \begin{bmatrix} N_i & F_i \\ -L_{Ii}C_a & 0 \end{bmatrix} e_a(t) \quad (14)$$

where:

$$e_a(t) = \begin{bmatrix} e(t) \\ \tilde{q}(t) \end{bmatrix} \quad (15)$$

Using (98), equation (14) can be written as follows:

$$\dot{e}_a(t) = \sum_{i=1}^{M} h_i(x(t))(\overline{A}_i - \overline{K}_i \overline{C})e_a(t) \quad (16)$$

with:

$$\overline{A}_i = \begin{bmatrix} PA_{ai} & F_i \\ 0 & 0 \end{bmatrix}, \ \overline{K}_i = \begin{bmatrix} K_i \\ L_{Ii} \end{bmatrix}, \ \overline{C}_a = [C_a \quad 0]$$

The convergence conditions of the state estimation error and unknown inputs are obtained by using the following Lyapunov quadratic function:

$$V(e(t)) = e_a(t)^T X e_a(t), \quad X = X^T > 0 \quad (17)$$

Its dynamic can be expressed as:

$$\dot{V}(e(t)) = \dot{e}_a(t)^T X e_a(t) + e_a(t)^T X \dot{e}_a(t)^T \quad (18)$$

Using (16), $\dot{V}(e(t))$ can be written as:

$$\dot{V}(e(t)) = \sum_{i=1}^{M} h_i(x(t))[e_a(t)^T(\overline{A}_i^T X + X\overline{A}_i \\ - \overline{C}^T \overline{K}_i^T X - X\overline{K}_i \overline{C}]e_a(t) \quad (19)$$

The derivative of the Lyapunov function is negative if:

$$\overline{A}_i^T X + X\overline{A}_i - \overline{C}^T \overline{K}_i^T X - X\overline{K}_i \overline{C} < 0, \quad \forall i \in \{1, \cdots, M\} \quad (20)$$

In order to solve the conditions (20) with the classical LMI approaches, we consider the following change of variables:

$$W_i = X\overline{K}_i \quad (21)$$

Taking into account the change of variable (21) the expression (20) becomes:

$$\overline{A}_i^T X + X\overline{A}_i - \overline{C}^T W_i^T - W_i \overline{C} < 0 \quad (22)$$

The gain of the observer is computed by:

$$\overline{K}_i = X^{-1} W_i \quad (23)$$

The equation (22) allows formulating the following theorem:

Theorem 1: *The state and unknown inputs estimation error between the unknown input observer (6) and the TS system (5) converges asymptotically to zero if there exists a symmetric and positive definite Lyapunov X matrix $X = X^T > 0$ and matrices W_i for all $i \in \{1, \cdots, M\}$:*

$$\overline{A}_i^T X + X\overline{A}_i - \overline{C}^T W_i^T - W_i \overline{C} < 0, \quad i = 1, \cdots, M \quad (24)$$

The gain of the observer is computed by:

$$\overline{K}_i = X^{-1} W_i \quad (25)$$

To improve the observer performance (6), the dynamics of the TS fuzzy observers is chosen in such a way that, it is substantially faster than that of the TS fuzzy model (5).

The eigenvalues of the linear models must be fixed in a region $S(\alpha,\beta)$ defined in the complex plane as the intersection of the left half-plane bounded by a line of abscissa $(-\alpha)$ and a disk of centre $(0,0)$ and radius β.

The inequality (22) of the previous approach is replaced by solving the following inequality [5] :

$$\begin{cases} \begin{bmatrix} -\beta X & \overline{A}_i^T X - \overline{C}^T W_i^T \\ X\overline{A}_i - W_i\overline{C} & -\beta X \end{bmatrix} < 0 \\ \overline{A}_i^T X + X\overline{A}_i - \overline{C}^T W_i^T - W_i\overline{C} + 2\alpha X < 0 \end{cases} \tag{26}$$

The resolution of conditions (11) is carried out, as indicated above in three steps:

From the first equation (11), the matrix H is computed as follows:

$$\begin{bmatrix} P & H \end{bmatrix} = \begin{bmatrix} I_p \\ C_a \end{bmatrix}^T \left(\begin{bmatrix} I_p \\ C_a \end{bmatrix} \begin{bmatrix} I_p \\ C_a \end{bmatrix}^T \right)^{-1} \tag{27}$$

where I_p is an identity matrix of full rank.

After resolution of the inequality (24), the gains are determined by the equation:

$$\overline{K}_i = X^{-1} W_i \tag{28}$$

From second and third equations (11), it is possible to calculate:

$$\begin{cases} G_i = PB_{ai} \\ F_i = PE_{ai} \\ N_i = PA_{ai} - K_i C_a \\ L_{Pi} = K_i + N_i H \end{cases} \tag{29}$$

3. Unknown Input PMI Fuzzy Observer Synthesis

In this section, the d^{th} derivative for the unknown input is null and it's assumed to be a bounded time varying signal:

A4. $q^{(d)}(t) = 0$

Generally, the unknown input satisfying the hypothesis **(A4)** cannot be estimated with a good precision with the using of UI-PI observer which requires the condition that the unknown input is constant ($\dot{q}(t) = 0$).

Then, unknown input PMI observer is more required and more adequate for such problem, because the estimation of the $(d-1)^{th}$ derivatives of the unknown input and gives a good precision [23].

Consider the unknown input proportional multiple integrals observer for T-S systems:

$$\begin{cases} \dot{z} = \sum_{i=1}^{M} h_i(x)\left[N_i z + G_i u + F_i \hat{q} + L_{Pi} y\right] \\ \hat{x}_a = z + Hy \\ \hat{y} = C_a \hat{x}_a \\ \dot{\hat{q}}_0 = \sum_{i=1}^{M} h_i(x) L_{Ii}^0 (y - \hat{y}) + \hat{q}_1 \\ \dot{\hat{q}}_1 = \sum_{i=1}^{M} h_i(x) L_{Ii}^1 (y - \hat{y}) + \hat{q}_2 \\ \vdots \\ \dot{\hat{q}}_{d-2} = \sum_{i=1}^{M} h_i(x) L_{Ii}^{d-2} (y - \hat{y}) + \hat{q}_{d-1} \\ \dot{\hat{q}}_{d-1} = \sum_{i=1}^{M} h_i(x) L_{Ii}^{d-1} (y - \hat{y}) \end{cases} \tag{30}$$

where:

$\hat{q}_i,\ i = 1,2,\ldots,(d-1)$ are the estimation of the $(d-1)$ first derivatives of the unknown input $q(t) = 0$.

The state and unknown inputs estimation errors are:

$e = x - \hat{x}$, $e_0 = \dot{q} - \dot{\hat{q}}_0$, $e_{d-1} = \dot{q}_{d-1} - \dot{\hat{q}}_{d-1}$

The dynamics are given in the following form:

$$\begin{cases} \dot{e} = \sum_{i=1}^{M} h_i(x)\left[(PA_{ai} - K_{Pi} C_a)e + F_i e_0\right] \\ \dot{e}_0 = \sum_{i=1}^{M} h_i(x)\left[-K_{Ii}^0 C_a e + e_1\right] \\ \dot{e}_1 = \sum_{i=1}^{M} h_i(x)\left[-K_{Ii}^1 C_a e + e_2\right] \\ \vdots \\ \dot{e}_{d-2} = \sum_{i=1}^{M} h_i(x)\left[-K_{Ii}^{d-2} C_a e + e_{d-1}\right] \\ \dot{e}_{d-1} = \sum_{i=1}^{M} h_i(x)\left[-K_{Ii}^{d-1} C_a e\right] \end{cases} \tag{31}$$

The equations (31) can be rewritten in augmented form as following:

$$\dot{\tilde{e}} = \sum_{i=1}^{M} h_i(x)\left[(\tilde{A}_i - \tilde{K}_i \tilde{C})\tilde{e}\right] \tag{32}$$

where:

$$\tilde{e} = \begin{bmatrix} e \\ e_0 \\ e_1 \\ \vdots \\ e_{d-2} \\ e_{d-1} \end{bmatrix}, \quad \tilde{A}_i = \begin{bmatrix} PA_{ai} & F_i & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_s & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & I_s \\ 0 & 0 & 0 & & 0 & 0 \end{bmatrix},$$

$$\tilde{K}_i = \begin{bmatrix} K_{Pi} \\ K_{Ii}^0 \\ K_{Ii}^1 \\ \vdots \\ K_{Ii}^{d-2} \\ K_{Ii}^{d-1} \end{bmatrix} \quad \tilde{C} = [C_a \quad 0 \quad 0 \quad \cdots \quad 0 \quad 0]$$

In the following, we are only interested with particular component $e(t)$ and $e_0(t)$ of $\tilde{e}(t)$:

$$\begin{bmatrix} e \\ e_0 \end{bmatrix} = \bar{C}\tilde{e}$$

where:

$$\bar{C} = \begin{bmatrix} I_n & 0 \\ 0 & I_s \end{bmatrix} \bar{C} = \left[\begin{matrix} I_n & 0 \\ 0 & I_s \end{matrix} \middle| \begin{matrix} 0 & \cdots & 0 \end{matrix}\right]$$

0: represents null matrix with appropriate dimensions.

Theorem 2: *The estimation error of the state and the unknown inputs between the unknown input observer (30) and the TS system (5), converge asymptotically to zero if there exists a symmetric and positive definite Lyapunov* X *matrix* $X = X^T > 0$ *and matrices* W_i *for all* $i \in \{1, \cdots, M\}$*, such as:*

$$\tilde{A}_i^T X + X\tilde{A}_i - W_i\tilde{C} - \tilde{C}W_i^T < 0, \quad i = 1,\ldots,M \tag{33}$$

The gains of the observer are derived from:

$$\tilde{K}_i = X^{-1}W_i \tag{34}$$

Proof: The proof of the theorem 2 is similar to the proof of the theorem 1 by using the system (32) .

In the case when **A4** is not satisfied i.e. $q^d(t)$ is bounded but then, we can consider the d^{th} derivative of $q(t)$ as a perturbation (unknown input).

The additional component q_d is added in the state vector.

4.Simulation results

Consider the nonlinear system consisting of an arm actuated by a DC motor (see Figure 1), proposed in [29]:

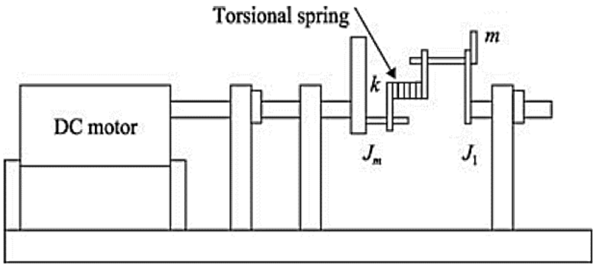

Figure 1. Manipulator arm actuated by DC motor

The mathematic model of the Manipulator arm actuated by DC motor is giving by the following equations:

$$\begin{cases} \dot{\theta}_m(t) = \omega_m(t) \\ \dot{\omega}_m(t) = \dfrac{k}{J_m}(\theta_1(t) - \theta_m(t)) - \dfrac{B}{J_m}\omega_m(t) + \dfrac{k_t}{J_m}u(t) \\ \dot{\theta}_l(t) = \omega_l(t) \\ \dot{\omega}_l(t) = -\dfrac{k}{J_l}(\theta_1(t) - \theta_m(t)) - \dfrac{mgh}{J_l}\sin(\theta_l(t)) \end{cases} \tag{35}$$

where:

$\theta_m(t)$ represents the angular position of the motor,

$\omega_m(t)$ its angular velocity,

$\theta_1(t)$ is the angular position of the arm,

$\omega_l(t)$ is the angular speed of the arm.

The input of the system in this case is considered as a step signal, and the initial conditions are $[0 \quad 0 \quad 0 \quad 0]^T$.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -1.95 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}$$

$$x = \begin{bmatrix} \theta_m \\ \omega_m \\ \theta_1 \\ \omega_1 \end{bmatrix}, \quad f(x) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33\sin(x_3) \end{bmatrix}, \quad E = \begin{bmatrix} 0.5 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Using the nonlinear sector transformation approach [23], a TS model representing exactly the behavior of the model (35) is obtained as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{M} h_1(x(t))[A_i x(t) + B_i u(t) + E_i q(t)] \\ y(t) = Cx(t) \end{cases} \tag{36}$$

with:

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -22.83 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -18.77 & 0 \end{bmatrix}, B_1 = B_2 = B$$

The dynamic of the system is affected by the unknown inputs vector $q(t)$ and the weighting functions depend on the first component $x_3(t)$ of the state vector *x* and are defined as follows:

$$\begin{cases} \mu_1(z(t)) = \dfrac{z(t) + 0.2172}{1.2172} \\ \mu_2(z(t)) = \dfrac{1 - z(t)}{1.2172} \end{cases}$$

where:

$$z(t) = \frac{\sin(x_3)}{x_3}$$

According to the theorem 1, a PI observer is synthesized taking account the unknown inputs given by: *q*(*t*), which is the time varying signals with the negligence of the fifth derivatives. Also, a PMI observer is designed and synthesized according to the theorem 2, with $d = 5$.

The initial conditions of the Takagi-Sugeno model are: $x(0) = [1 \;\; 0.5 \;\; -0.5 \;\; 0.25]$, and the initial conditions of the observers are null.

The obtained simulation results are depicted in the following figures:

A. Case 1 : Hypothesis (A3) is satisfied

Figure 2 presents the state estimation of the system using the UI-PI observer.

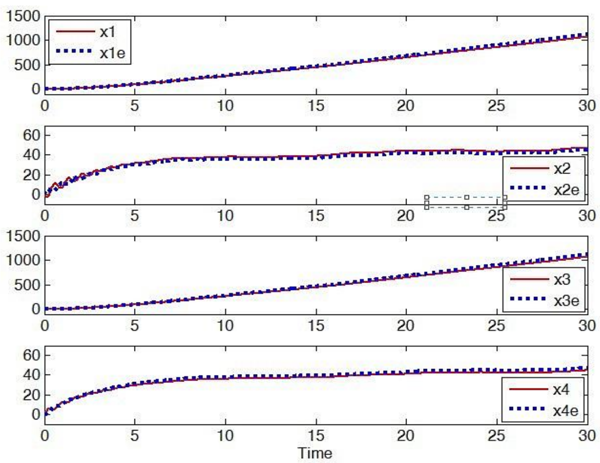

Figure 2. State estimation with UI-PI observer

It can be seen from figure 2 that the proposed UI-PI observer provides an acceptable state estimation.

Figure 3 shows the state estimation error using the UI-PI observer.

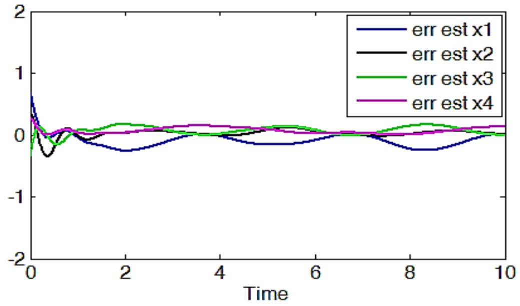

Figure 3. State estimation error with UI-PI observer

The simulation results obtained in Figure 3 show a less quality in the state estimation of the considered system; this is due to the non-satisfaction of hypothesis (A3).

Figure 4 shows the unknown input and its estimation obtained by using a UI-PI observer.

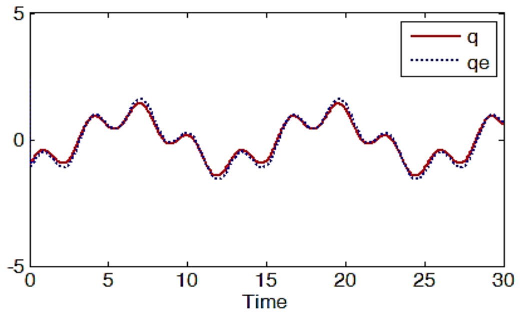

Figure 4. Unknown input estimation with UI-PI observer

The simulation results obtained in figure (4) show a less quality in the unknown input estimation of the considered system; this is due to the non-satisfaction of hypothesis (A3).

Figure 5 shows the state estimation using the UI-PMI observers.

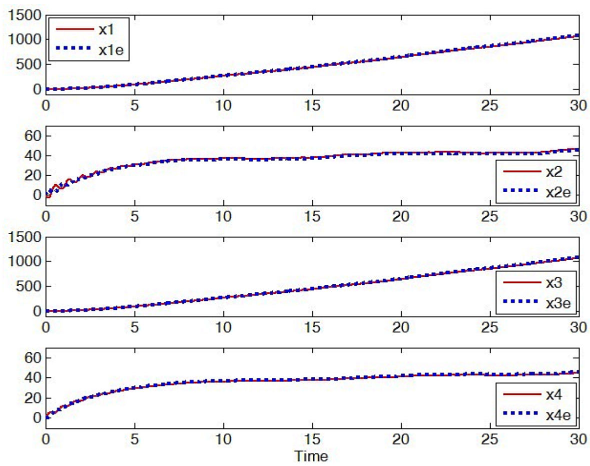

Figure 5. State estimation with UI-PMI observer

The results of state estimation obtained by using the UI-PMI presented in the figure (5) are better compared by the results given by using the UI-PI observer.

Figure (6) presents the state estimation error using the UI-PMI observer.

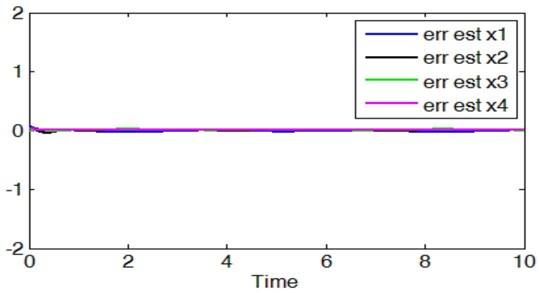

Figure 6. State estimation error with UI-PMI observer

In the general case when hypothesis (A4) is satisfied, the simulation results obtained in figure (6) using UI-PMI observer show a better quality in the state estimation of the considered system compared with the results given by the UI-PI observer.

Figure 7 shows the unknown input and its estimation obtained by using the UI-PMI observer.

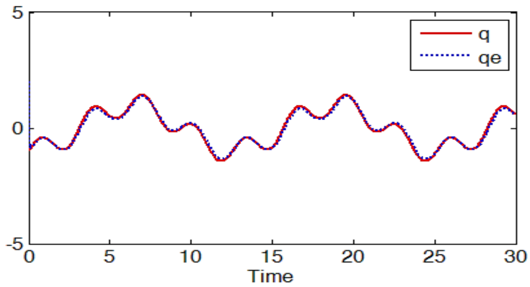

Figure 7. Unknown input estimation with UI-PMI observer

In the general case the hypothesis (A4) is satisfied, the simulation results show that the unknown input estimation obtained in figure (7) are better compared to the results given by using the UI-PI observer of the considered system.

Figure 8 shows the unknown input estimation error obtained using the UI-PI observer and the UI-PMI observer.

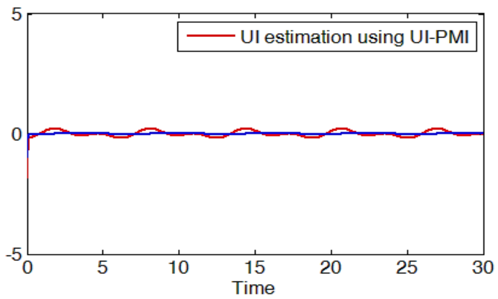

Figure 8. The unknown input estimation error obtained with UI-PI and UI-PMI observers.

The simulation results in figure (7) show that the unknown input estimation using UI-PMI observer is better compared to the unknown input estimation given by the UI-PI observer.

5. Conclusion

In this paper, two new approaches have been presented, for the synthesis of unknown input proportional integral (UI-PI) observer and unknown input proportional multiple integral (UI-PMI) observer.

The developed observers are dedicated for simultaneously state and unknown input estimation of nonlinear TS augmented systems.

The developed unknown input proportional integral observer is interesting for the simultaneously estimation of state and constant or slowly varying unknown inputs compared to the unknown input proportional multiple integral observers.

The UI-PMI observer is a good way to obtain more precise estimation of states and unknown inputs in the general case.

The convergence conditions of the state estimation error are given in the LMI formulation

6. Bibliographic references

[1] M. Darouach, M. Zasadzinsky, et S.J. Xu, “Full-order observers for linear systems with unknown inputs”, *IEEE Transactions on Automatic Control*, 93(3), March 1**4, pp:606-609.

[2] M. Didier, B. Gaddoune, J. Ragot, « Estimation of unknown inputs in linear systems », *American Control Conference*, ACC’94, Baltimore, United States. 1, Jun 1994, pp:1195-1197.

[3] B. Marx, D. Koenig, and D. Georges, “Robust fault diagnosis for linear descriptor systems using proportional integral observers”, *Decision and control*, 2003.

[4] M. Darouach, “Complements to full order observer design for linear systems with unknown inputs”, *Applied Mathematics Letters*, Vol. 22, Issue 7, July 2009, pp:1107-1111.

[5] A. Akhenak, *Conception d’observateurs non linéaires par approche multi-modèle: application au diagnostic*, Thèse de doctorat, Institut National Polytechnique de Lorraine, France, 2004.

[6] R. Orjuela, B. Marx, J. Ragot, and D. Maquin, “On the simultaneous state and unknown inputs estimation of complex systems via a multiple model strategy”, *IET Control Theory and Applications*, 3(7), 2009, pp:877-890.

[7] W. Zhao, H. Li, L. Zou, and W. Huang, “Nonlinear unknown input observer based on singular Value decomposition aided reduced dimension cubature Kalman filter”, *Mathematical Problems in Engineering*, vol. 2017, Article ID 1267380, 13 pages, 2017.

[8] R. Murray-Smith, and T.A. Johansen, *Multiple model approaches to modelling and control*, Taylor and Francis, London, 1997.

[9] R. Orjuela, B. Marx, J. Ragot, M. Didier, « Conception d’un observateur PI pour un multi-modèle découplé », *8e Conférence internationale des sciences et techniques de l’automatique*, STA’2007, Sousse, Tunisie, 2007, pages 1 :10.

[10] R. Orjuela, B. Marx, J. Ragot, and D. Maquin, “PI observer design for discrete-time decoupled multiple models”, *5th Workshop on advanced*

control and diagnosis, ACD 2007, Nov. 2007, Grenoble, France, CDROM, 2007.

[11] H. Hamdi, M. Rodrigues, C. Mechmeche and N. Benhadj Braiek, "Observer based fault tolerant control for Takagi-Sugeno nonlinear descriptor systems", *Proc. Engineering and Technology* vol. 1, 2013, pp:52-57.

[12] I. Elleuch, A. Kheder, and K. Ben Othmen, "Sensor fault estimation using proportional integral observer based on sliding mode principle for uncertain Takagi-Sugeno fuzzy systems", *Proc. of engineering and technology* (PET), 2016, pp:213-219.

[13] A. Akhenak, M. Chadli, J. Ragot, D. Maquin, "State estimation via multiple observer with unknown input. Application to the three tank system", *5th IFAC Symposium fault detection supervision and safety for technical processes, Safe process*, pp:245-251, Washington, US, June 9-11, 2003.

[14] S. Bezzouacha, B. Marx, D. Maquin, and J. Ragot, "On the unknown input observer design: a decoupling class approach", *9th IEEE International conference on control and Automation*, ICCA11, Santiago Chile, December 19-21, 2011.

[15] D. Ichalal, M. Benoît, M. Didieer, J. Ragot. *State estimation of system with bounded uncertain parameters: interval multi-model approach. International journal of adaptive control and signal processing*, Wiley, 2018, 32(3), pp.480-493.

[16] D. Koenig, and S. Mammar, "Design of proportional-integral observer for unknown input descriptor systems", *IEEE Transactions on Automatic Control*, vol. 47, N°.12, December 2002.

[17] P. Bergsten, R. Palm, and D. Driankov, "Observers for Takagi-Sugeno fuzzy systems", *IEEE Transactions on Systems, Man, and Cybernitcs-Part B: Cybernetics*, 32(1), 2002, pp:114-121.

[18] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, "Linear matrix inequalities in systems and control theory", *SIAM studies in applied mathematics*, vol. 15, SIAM, Philadelphia, 1994.

[19] M. Chadli, *Analyse des systèmes non linéaires décrits par des multi-modèles*, Thèse de doctorat, Institut National Polytechnique de Lorraine, 2002.

[20] J. Korbicz, M. Witczak, and V. Puig, "LMI-based strategies for designing observers and unknown input observers for nonlinear discrete-time systems", *Bulletin of the Polish Academy of Sciences, Technical Sciences*, 55(1), 2007, pp:31-42.

[21] T. Youssef, M. Chadli, and M. Zelmat, "Synthesis of an unknown inputs proportional integral observer for TS fuzzy models", *European control conference* (ECC), July 17-19, 2013, Zürich, Switzerland.

[22] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Conception d'observateurs pour un modèle de Takagi-Sugeno à variables de decision non mesurables", *11th International Conference on Sciences and Technics of Automatic control*, STA'2007, Sousse, Tunisia, November, 05-07,2007.

[23] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Simultaneous state and unknown inputs estimation with PI and PMI observers for Takagi-Sugeno model with unmeasurable premise variables", In *17th Mediterranean Conference on Control and Automation*. MED, 2009.

[24] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Multi-observateurs à entrées inconnues pour un système de Takagi-Sugeno à variables de decision non mesurables", *Revue Électronique de la SEE*, 2009, vol. 6, pp :9-15.

[25] M. Share Pasand, M. Aliyari Sh., and H.D. Taghirad, "Unknown input-proportional integral observer for singular systems: Application to fault detection", *Proc. of ICEE*, 2010, 978-1-4244-6761-7/10.

[26] W. Jamel, A. Kheder, N. Bouguila and K. Ben Othman, "Observer design for simultaneous state and faults estimation", *International journal on computer science and engineering* (IJCSE), vol. 5, No.09 Sept. 2013.

[27] Z. Gao, and D. Ho, "Proportional multiple-integral observer design for descriptor systems with measurement output disturbances", *IEEE proceeding control theory and application*, vol. 151, NO. 3, 2004, pp:279-288.

[28] W. Jamel, A. Kheder, N. Bouguila and K. Ben Othman, "Design of unknown inputs and multiple integral observers for Takagi-Sugeno multiple model", *12th International multi-conference on systems, signals and devices* (SSD), March 2015.

[29] R. Rajamani, and Y.M. Cho, "Existence and design of observers for nonlinear systems: relation to distance to unobservability", *International journal of control*, 69(5), 1998, pp: 717-731.

Funding Sources

This work is sponsorship by (DGRSDT) The General Directorate of Scientific Research and Technological Development, Algeria.

Authors' Biographies

Abdelhani DJEDDI was born in Algeria in 1973, he received the State Engineer degree 20 1996 on Control.
He was an active industrial header at the Algerian Company of Electrical and Gaz "SONELGAZ" from 1999 to 2013.
He was the responsible of several electrical projects.
He has obtained his Magister degree in 2007 on Control and Automatic and the PhD on Automatic in 2017 from University Badji Mokhtar Annaba.
He is an associate professor in electrical engineering at the Department of Electrical Engineering, Faculty of Sciences and Technology, University Lari Tebessi, Tebessa, Algeria.
His research area of interests includes the modelling and control in industrial systems, the diagnosis and new reliability engineering, fault detection and isolation in industrials process, intelligent system based on fuzzy logic.
He is the founder of the International Conference on Sustainable, Renewable Energy Systems and Applications (ICSRESA).
e-mail: abdelghani.djeddi@univ-tebessa.dz

Youcef SOUFI was born in Tebessa, Algeria. He received a B.Sc. degree and Doctorate degrees from the University of Annaba, Algeria, in 1991 and 2012 respectively, and a Magister degree in 1997 in Electrical Engineering from Tebessa University, Algeria.
Currently, he is a Professor in the department of Electrical Engineering, Faculty of Sciences and Technology, Larbi Tebessi University, Tebessa, Algeria.
His research interests include electrical machines, diagnostics, wind and solar energy, power electronics and drives applied to renewable and sustainable energy.
e-mail address is: y_soufi@yahoo.fr

Salah Chenikher received his Magister and Doctorate from the Institute of Electronics, Annaba University in 1996 and 2007, respectively.
He is currently a Professor and researcher in the University of Larbi Tebessi, Tebessa. He is Currently, he is the Director of LABGET Laboratory, Larbi Tebessi University.
He has participated in several international research projects and has led several national research projects. His research interests include control of renewable energy systems, fault tolerant control and diagnosis.
He is acting as an expert in several national and internationals commissions and collaboration research activities.
He has published many national/international conferences and journals papers.
e-mail address is: s_chenikher@yahoo.fr

Abdelaziz **Aouiche** received his M.Sc. from the university of M'sila, Algeria in 2002 and 2006, respectively all in electrical engineering.
He received the Ph.D degree from university of Batna, Algeria, in 2016. He is currently an associate professor
Department of Electrical Engineering, University Larbi Tébessi, Tébessa-Algeria.
His main research interests are in contol of nonlinear systems, neuro-control systems and fuzzy control systems, Antenna and Computational electromagnetics, planar and cylindrical microstrip antennas and transmission lines, neural networks and fuzzy logic, integration of fuzzy-neural systems.
E mail address is: abdelazizaouichester@gmail.com