Optimal PMUs Placement to ensure Power System Observability under Various Contingencies

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Abstract

The PMUs are high-precision devices used to monitor the operation of the power system. These devices provide synchronized measurement of AC voltage and current phasors at the same time by the global positioning system (GPS). Due to the high costs of these devices, they cannot be placed on all network buses. Therefore this paper aims to find the lowest possible number of PMUs to ensure a complete observability of the power system and improve the measurement redundancy, During normal cases (with and without ZIBs), Then in conditions of failure of single PMUs and outage of single line, by using gravitational search algorithm (GSA). The suggested GSA has been applied to the IEEE 14-bus, 24-bus, 30-bus, 57-bus, New England 39-bus, and Algerian 114-bus. To check the effectiveness and robustness of this technique, a comparison of results is made with some of the methods recently reported. Comparison results show that the proposed technique has succeeded in significantly reducing the number of PMUs and improving the redundancy of measurements.

Keywords: Phasor measurement units (PMUs), Zero injection bus (ZIB), measurement redundancy, gravitational search algorithm (GSA).

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1. Introduction

Recent years have seen an increase in electricity demand caused by industrialization, urbanization and population growth. All these factors lead to the considerable and rapid extension of the size and complexity of network.

Moreover, the electrical networks are often exposed to contingencies that can cause partial or total failures.

There is a wide variety of such contingencies: line failures, sudden variations in load, short-circuit faults, lightning, etc. Depending on their severity, they can lead to the loss of network synchronization, damage to the various protection and transmission equipment etc. This can negatively affect the reliability and safety of the network.

As a recent example, it should be remembered the biggest blackout in United States in 2003, which was caused by a lack of real-time data collection through the event.

The two-day disaster reached 55 million people and generated a massive economic loss estimated at around US$4-10 billion [1].

Indeed, the operators of electrical networks must use high-performance tools to predict instability to ensure the continuity of service, security and reliability of the system.

In the classical version, the control is made based on the measurements collected by the SCADA system (Supervisory Control and Data Acquisition).

This system uses asynchronous and relatively slow means for collecting measurements, and consequently the rapid and dynamic phenomena of the power system are not effectively captured [2].

In addition, the SCADA system data does not contain the precise bus voltage phase angle and line currents [3].

At the beginning of 1990, Phasor measurement units (PMUs) were launched as devices with the capacity to measure real-time synchronous voltage and current phasors in electric power systems [4].

PMU has become an interesting solution because it can be used as a measuring tool that can provide synchronized phasor measurements, and progressively replace the role of standard SCADA measurement system in state estimation and become the foundation of future smart grid infrastructure [5].

Nowadays, with the advent of synchro-phasor technology, phasor measurement units (PMUs) at strategic busses on the power grid can provide synchronized voltage and current phasors in real time.

Due to the superior quality of these measurements, specifically their accuracy (30 to 60 data points per second), they provide a global observability of the network in real time. These data can be integrated into an operational environment and exploited to predict rapidly if the network is tending to become unstable or not. It offer time-synchronized (real-time) phasor measurements in power systems, it is obtainable with the Global Positioning System (GPS), which has an accuracy of less than 1 μs [6].

To state a power system is fully observable when the voltage Phasor of all busses can be determined both directly, or indirectly. It's essential to find the optimal placement to maximize the measurement redundancy [7] and reduce the cost of PMUs itself.

From another side, it is not essential to place PMUs in all busses because the bus voltage phasor, which is next to the PMU located bus, can be calculated by the parameter of branch and the measurement of branch current phasor.

Lately, various optimization approaches have been suggested to resolve the OPP problem. To find the minimum number and the optimal placement of PMUs in electrical power system to achieve a complete observability.

There are two essential groups of optimization method Mathematical and Metaheuristic algorithms.

The first group consists of the integer linear programming (ILP). It is the commonest optimization method utilized to resolve the OPP problem as in [8], [9] and [10].

A sequential quadratic programming (SQP) in [11], a binary semi-definite programming (BSDP) [12].

Recently, Metaheuristic methods have been used for resolving the problem of OPP such as genetic algorithm (GA) [13], [5] and simulated annealing (SA) [14]and [15], Differential evolution (DE) [16] and[17].

In [18] a new DE approach based on Pareto non-dominated sorting was suggested to solve a multi-objective problem, and particle swarm optimization (PSO) through a discreet problem known as (BPSO) [19] and [20].

In modern times, the importance of the measurement redundancy (SORI) has been outlined among the most important parameters to secure controlling the power systems networks [11]. In order to estimate the quality of every PMU placements set, which has the greatest number of SORI signifies the PMUs placement set has a superior quality solution and more reliable for possible contingencies that compared to the PMUs placement set which has a low number of SORI.

The mains purpose of this paper is to demonstrate the effectiveness of GSA for resolving the optimal PMUs placement issue considering four cases: base case, ZIBs, single PMU failure and outage of single line for full monitored network.

The rest of this article is arranged as following: section 2 presents the mathematical formulation of PMU placement problem with and without ZIBs and topological observability rules related to each of them, loss of one PMU and single line outage.

Section 3 specifies a brief idea of GSA technique. In section 4 the Implementation of GSA to resolve the OPP problem is given. Lastly, the simulation results are presented in section 5 and section 6 provides an article conclusion.

2. Formulation of the PMUs placement problem

Generally, the principle goal of the OPP issue is to obtain the minimum number of PMU required and their locations to reach a fully monitored power system.

So, the objective function of OPP problem is formulated as following:

$$Min\left\{\sum_{i=1}^{N_{bus}} x_i\right\} \quad (1)$$

Subject to $f_i \geq I$ (2)

Where :

$$f_i = \sum A_{ij} x_i \quad (3)$$

where:

N_{bus} is the number of busses,

f_i is the observability function of bus i,

I is a vector whose elements are ones.

A is a connectivity matrix, whose elements are defined as:

$$A_{ij} = \begin{cases} 1 & if\ i = j \\ 1 & if\ the\ bus\ i\ s\ connected\ to\ bus\ j \\ 0 & otherwise \end{cases} \quad (4)$$

X is a binary decision variable vector defined as:

$$x_i = \begin{cases} 1 & if\ a\ PMU\ is\ installed\ at\ bus\ i \\ 0 & otherwise \end{cases} \quad (5)$$

To assess the quality of every set of PMU placements, measurement redundancy concept of bus observability index (BOI) and system of redundancy index (SORI) are used.

In [20] redundancy measurement (SORI) is acquired as follow:

$$(M - BOI)^T \times (M - BOI) \quad (6)$$

Since BOI implies how many times each bus has been observed by the PMUs. M is the coveted value of measurement redundancy.

The vector $(M - BOI)$ calculates the difference between the coveted and current number of times the bus is observed.

So, X should be minimized, and AX need to be maximized. Two reasons are as following: minimizing the Eq. (1) and maximizing the inverse of Eq. (6).

A. PMU placement rules

Rule 1: A PMU-equipped bus, its voltage phasor and all branches currents adjacent to it are measured directly by the PMU, this is called direct measurement.

Rule 2: Knowing the voltage and its current phasor at one end of branch, the voltage phasor of the other end can be obtained just by ohm's law. This is called pseudo measurement.

Rule 3: Assuming a branch where the voltage phasors at both ends are known, the current of this branch can be obtained by applying ohm's law. This is called pseudo measurement.

To describe the work of these rules, deem Figure 1.

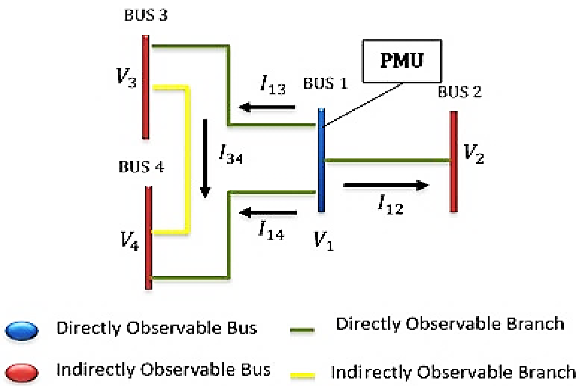

Figure 1. Modelling PMUs placement rule

where: the bus 1 is a PMUs installed bus, the value V_1, I_{1-2}, I_{1-3} and I_{1-4} can be obtained directly following the rule 1.

Since the value of current branches I_{1-2}, I_{1-3} and I_{1-4} are determined, so the voltages at bus 2, 3 and 4 can be computed using the ohm's law following the second rule.

According to the rule 3, the current of the branch $3-4$ will be obtainable.

B. Impact of zero injection bus (ZIB)

A zero injection bus is other coefficient that may decrease the number of PMUs necessary to attain a completely observed system. These types of busses are responsible to transmit power through the network transmitting lines without injecting or consuming it.

Thus, the sum of the branch currents connected with a ZIB is zero according to KCL.

The ZIB rules for evaluating the system observability as following:

Rule 1: When all neighbouring busses of an observable ZIB, are observables excluding only one, the non-observable bus may be defined to be observable by applying the KCL to ZIB.

Rule 2: when a group of observable busses neighbouring to unobservable ZIB, the ZIB may be identified as observable via the node equation.

To clarify these rules, consider Figure 2.

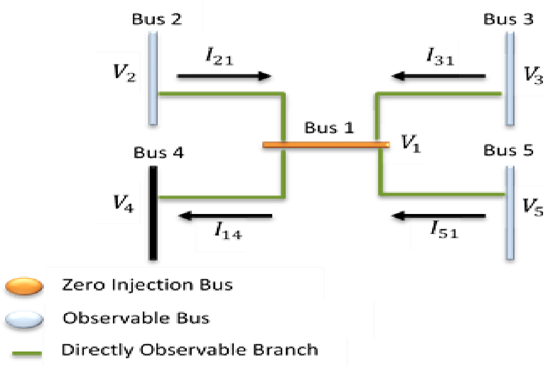

Figure 2. Modelling the rules of ZIBs

Where bus 1 is a ZIB and its adjacent buses is 2, 3, 4 and 5.

Consider the buses 1, 2, 3 and 5 are all observables (Their voltages are known) excepting the bus 4.

Following the rule 1, the current I_{1-4} may be calculated by applying KCL at bus 1.

For the last rule, consider that buses 2, 3, 4 and 5 are observable and that the ZIB is not observable.

Applying the node equation to ZIB, the voltage of bus 1 may be determined.

C. Single PMU failure

PMUs have become more efficient and reliable, but they are inclined to failure, just like as most electrical products, this is due to failure of microprocessor, communicational system, etc. If one of the PMUs failed, the bus that was observed by the PMU become unobservable if the bus does not have any additional PMUs monitoring it. In this case, it should be assured that if any single PMU fails, the network stays observable.

To handle this problem, every bus needs to be monitored by at minimum two PMUs (except the radial bus). The constraints is as follow:

$$\boldsymbol{f_i \geq 2} \tag{7}$$

D. Line outage contingency

Line outage is among the contingencies that could be unsafe to observability system. Outage of a branch can result increased loss of complete monitoring in among their terminals, which might be observable using this branch current phasor.

The following are the constraints in case of an outage of single line:

$$f_i^k = \geq 1 \qquad \forall\, i \in I, \forall\, k \in K \tag{8}$$

where

$$f_i^k = \sum_{j \in I} a_{ij}^k x_j, \quad \forall\, i \in I, \forall\, k \in K \tag{9}$$

The binary connectivity parameter in case where line k is out, is defined as follows:

$$a_{ij}^k = \begin{cases} 0 & \text{line k is between i} - \text{j} \\ a_{ij} & \text{Otherwise} \end{cases} \tag{10}$$

3. Gravitational search algorithm

This paper adopts a recent meta-heuristic optimization approach known as Gravitational Search Algorithm (GSA) as the perfect solution method to solve the optimization problem.

E. Rashedi developed this new algorithm in 2009 [21]. The GSA is inspired from the newton's theory which states: each mass attracts each other mass in the cosmos by a gravitational force that is proportional directly to the product of their masses, and inversely proportional to the square of the distance that separates them. In GSA, each agent (mass) has four parameters: the inertia mass, the position and its active and passive gravitational mass.

The mass position of an agent refers to a solution of the problem. Its inertial and gravitational masses are determined utilizing the fitness function. For each iteration, these solutions are updated and the very best fitness along using its corresponding agent is stored. In this algorithm, the condition of termination is described by a constant number of iterations, achieving it the algorithm terminates immediately.

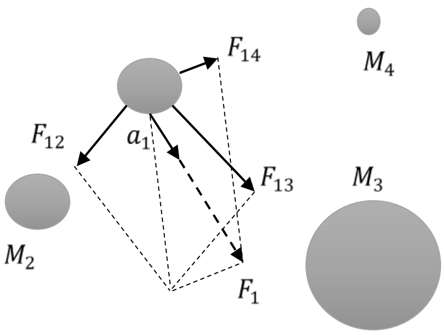

Figure 3. Acceleration and force vector

For more describing, the algorithm can be summarized as below:

Agents Initialization: Start the positions of the N number of agents arbitrarily.

$$X_i = (x_i^1, \dots, x_i^d, \dots, x_i^n) \quad for\ i = 1,2, \dots N \tag{11}$$

In which, x_i^d is the positioning of i^{th} agent in the d^{th} dimension and n is the size of the space of search.

Fitness development and best fitness computation for every agent: Execute for all agents the fitness evolution at every iteration.

Then, between all obtained results, the *worst* and *best* fitness in every iteration are considered as below:

$$worst(t) = \max_{j\in\{1,2,\dots N\}} fit_j(t) \tag{12}$$

$$best(t) = \min_{j\in\{1,2,\dots N\}} fit_j(t) \tag{13}$$

where, $fit_j(t)$represent the value of fitness of the agent j at iteration t, $best(t)$ and $worst(t)$ indicate the strongest and the weakness agent relating to their fitness value, respectively.

At iteration t, calculate the gravitational constant G:

$$G(t) = G_0 e^{-(\alpha t/T)} \tag{14}$$

In such issue, G_0has the value 100, α is defined to 10 and T as the number of total iterations.

Calculate the masses of the agents: using the equations below, determine inertia and gravitational masses for each agent at iteration t :

$$M_{ii} = M_{ai} = M_{pi} = M_i\ ; i = 1,2, \dots N \tag{15}$$

$$m_i(t) = \frac{fit_i(t) - worst_i(t)}{best(t) - worst(t)} \tag{16}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \tag{17}$$

In which, M_{ii} is the inertia mass of the agent i, $M_{ai}$and M_{pi} is the active and passive gravitational mass associated with the agent i, respectively.

Calculate the acceleration of the i^{th} agents at iteration t:

$$a_i^d = \frac{F_i^d(t)}{M_{ii}(t)} \tag{18}$$

$F_i^d(t)$ is the full force acting on i^{th} agent determined by the following equation:

$$F_i^d(t) = \sum_{j\in Kbest, j\neq i} rand_j F_{ij}^d(t) \tag{19}$$

$$F_{ij}^d(t) = G(t) \frac{M_{pi}(t) \times M_{aj}(t)}{R_{ij}(t) + \varepsilon} \left(x_j^d(t) - x_i^d(t) \right) \tag{20}$$

where $Kbest$ is the group of first K agents with the best fitness value and biggest mass and $rand_j$ is an arbitrary number in the range [0 1]. $F_{ij}^d(t)$Is the force functioning on agent i from agent j at iteration t, $R_{ij}(t)$ is the Euclidian distance between the two agents i and j at iteration t, $G(t)$ is the gravitational constant at iteration t. ε is a small constant.

Update positions and velocity of the agent at iteration $(t+1)$ as follow:

$$x_i^d(t+1) = x_i^d(t) + v_i^d(t+1) \tag{21}$$

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{22}$$

Reprise from steps 2-6 until iterations reach their maximal limit.

Return the best value of fitness calculated at last iteration as the global fitness and the positions of the related agent at specified dimensions as the global solution of that problem.

4. Application of the proposed technique

Gravitational Search Algorithm (GSA) is a stochastic optimization algorithm, it is based essentially on the law of gravity.

We begin by a primary population of solutions (masses) selected haphazardly.

We evaluate their relative performance (fitness); and generate a novel population of potential solutions by use the force attraction.

The decision variables for the OPP problem are PMU installation states.

Therefore, the structure of every agent (mass) of the GSA to resolve the OPP problem is as below:

$$X = [x_1, x_2, \dots, x_N] \tag{23}$$

The following is a summary of the algorithmic stages to resolve the OPP problem:

Step 1: Read line and bus data of the test network.
Step 2: Get the connectivity matrix (A) as presented in equation (4).
Step 3: Initialize the parameters of GSA: T, N, G_0 and α.
Step 4: Determine the space of search.
Step 5: Initialize population between lower and upper control variables values.

Step 6: For every agent in the population the value of fitness are computed for the problem of OPP.
Step 7: Update $worst(t)$, $best(t)$, $G(t)$ and $M_i(t)$ for each agent according to the fitness value.
Step 8: Compute the total force in various directions.
Step 9: Adjust acceleration a_i^d for $i = 1,2,\ldots N$ using Eq. (18).
Step 10: Updating the agent's positions and velocity utilizing Eq. (21) and Eq. (22).
Step 11: Reprise Steps 6-10 unto reach the criterion of termination.
Step 12: Terminate.

The basic principle of GSA is presented in the Figure 4.

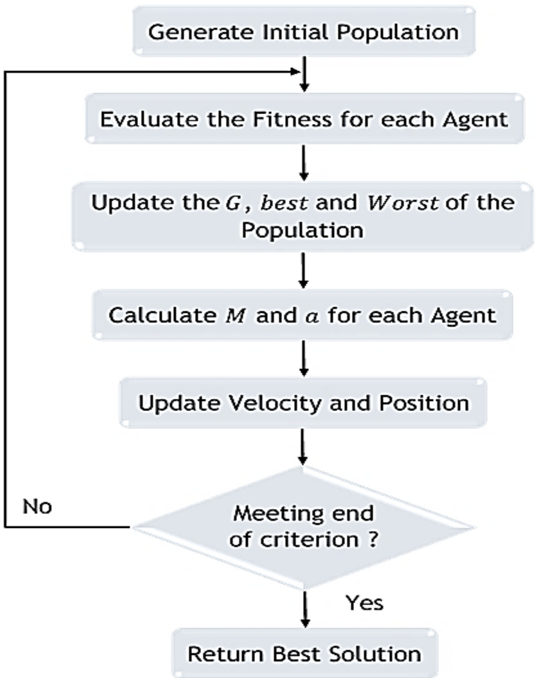

Figure 4. General Principle of GSA

5. Results and discussion

This paper applies the GSA to resolve the OPP problem.

By setting up PMUs in strategic buses, the network system become observable with the least number of PMUs.

The suggested PMU placement algorithm is implemented to various IEEE bus systems which range from 14-bus to 57-bus and Algerian 114-bus system for 4 simulation cases:

Case 1: Normal operating (Ignoring ZIBs).
Case 2: Considering ZIBs.
Case 3: Single PMU failure.
Case 4: Single line outage.

The simulations are executed by using MATLAB R2017b software.

The technical specification of the laptop used to perform the simulations is Intel core i7 2.4 GHz and 8 GB of RAM.

All simulation outcomes were reached assuming that every PMU contains as many channels number as possible and all PMUs have an equal cost.

The main aim of the suggested method is to find the least possible number of PMUs, which allows the system to be fully monitored while maximising measurement redundancy (SORI).

The optimal PMUs placement set, which has the greatest SORI is best compared to the set, which has low SORI.

A greater value of SORI attained implies that the monitoring system is too reliable for possible contingencies.

Table 1 displays the different data for each test systems.

Table 1. Specifications of test systems

Test System	Number of lines	Number of ZIBs	Location of ZIBs	Number of radial buses	Locations of radial bus
IEEE 14-Bus	20	1	7	1	8
IEEE 24-Bus	38	4	11, 12, 17, 24	1	7
IEEE 30-Bus	41	6	6, 9, 22, 25, 27, 28	3	11, 13, 26
NE 39-Bus	48	12	1, 2, 5, 6, 9, 10, 11, 13, 14, 17, 19, 22	9	30, 31, 32, 33, 34, 35, 36, 37, 38
IEEE 57-Bus	80	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48	1	33
Algerian network 114-Bus	175	22	2, 14, 16, 18, 27, 28, 31, 42, 44, 46, 48, 58, 60, 64, 72, 74, 75, 81, 86, 93, 96, 105	20	2, 14, 16, 18, 27, 28, 31, 42, 44, 46, 48, 58, 60, 64, 72, 74, 75, 81, 86, 93, 96, 105

Table 2 presents the number of PMUs (N_{PMU}), its places and the measurement redundancy value (SORI) for every test system, from this table, it can be noted that as the power system size augmented, the number of PMUs necessary that may attain complete observability increases.

Table 2. Number and position of PMUs in normal operating case

Test System	Number of PMUs; N_{PMUs}	Positions of PMUs	SORI
IEEE 14-Bus	4	2, 6, 7, 9	19
IEEE 24-Bus	7	2, 3, 8, 10, 16, 21, 23	31
IEEE 30-Bus	10	2, 4, 6, 9, 10, 12, 15, 19, 25, 27	52
NE 39-Bus	13	2, 6, 9, 10, 13, 14, 17, 19, 20, 22, 23, 25, 29	52
IEEE 57-Bus	17	1, 4, 9, 15, 20, 24, 25, 28, 29, 32, 36, 38, 39, 41, 46, 50, 53	71
Algerian network 114-Bus	33	2, 4, 13, 16, 18, 19, 20, 26, 29, 31, 34, 41, 42, 46, 47, 54, 57, 59, 63, 68, 69, 74, 82, 87, 88, 93, 96, 100, 101, 103, 105, 109, 111	159

For the case 2 (with ZIBs): the placement of PMUs and its SORI value is presented in Table 3.

Table 3. Number and position of PMUs when considering ZIBs

Test System	Number of PMUs; N_{PMUs}	Positions of PMUs	SORI
IEEE 14-Bus	3	2, 6, 9	16
IEEE 24-Bus	6	2, 8, 10, 15, 20, 21	29
IEEE 30-Bus	7	2, 4, 10, 12, 15, 18, 27	41
NE 39-Bus	8	8, 13, 16, 18, 20, 23, 25, 29	43
IEEE 57-Bus	11	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	60
Algerian network 114-Bus	28	2, 3, 4, 13, 18, 19, 20, 26, 29, 31, 34, 41, 47, 54, 57, 59, 63, 68, 69, 82, 88, 93, 98, 100, 102, 103, 109, 111	149

From Table 2 and Table 3, it is obviously that the required number of PMUs is minimized when there are ZIBs in the power system.

For instance, consider the necessary number of PUMs in Algerian 114-bus system, it should be noted that 33 PMUs are required to ensure full observability when ZIBs are ignored.

While the needed number of PMUs is minimized to 28 when the impact of ZIBs are present. That is possible because applying the rule 1 and 2 (concerning the existence of ZIBs) can reduce the number of buses that should be directly observed by the PMUs.

For the purpose of the effectiveness estimation concerning the suggested approach, the simulation outcomes under normal operation and considering Zero-Injection bus, which have been obtained using the suggested method are compared with previous researches in table 4.

Table 4 Comparison results with available techniques under normal operating case

Algorithm	Parameter	IEEE 14-Bus	IEEE 24-Bus	IEEE 30-Bus	NE 39-Bus	IEEE 57-Bus	Algerian network 114-Bus
Proposed Method	N_{PMUs}	4	7	10	13	17	33
	$SORI$	19	31	52	52	71	159
GWO [3]	N_{PMUs}	4	-	10	-	-	38
	$SORI$	19	-	48	-	-	157
MFO [3]	N_{PMUs}	4	-	10	-	-	39
	$SORI$	19	-	46	-	-	153
CS [3]	N_{PMUs}	4	-	10	-	-	37
	$SORI$	19	-	50	-	-	168
WDO [3]	N_{PMUs}	4	-	10	-	-	45
	$SORI$	19	-	44	-	-	185
VNS [22]	N_{PMUs}	4	7	10	-	17	-
	$SORI$	19	31	67	-	71	-
PSO-GSA [23]	N_{PMUs}	4	-	10	-	18	-
	$SORI$	19	-	52	-	77	-
BPSO [24]	N_{PMUs}	4	7	10	13	-	-
	$SORI$	19	31	48	49	-	-
PSO-MFO [25]	N_{PMUs}	4	-	10	-	17	-
	$SORI$	19	-	52	-	67	-
FPA [26]	N_{PMUs}	4	-	10	13	17	-
	$SORI$	19	-	52	52	72	-
BGSA [27]	N_{PMUs}	4	-	10	-	-	-
	$SORI$	19	-	52	-	-	-
MICA [28]	N_{PMUs}	4	-	10	-	17	-
	$SORI$	17	-	48	-	67	-
BSDP [12]	N_{PMUs}	4	-	10	-	17	-
	$SORI$	16	-	50	-	66	-
SQP [11]	N_{PMUs}	4	7	10	13	17	-
	$SORI$	19	31	48	52	71	-

Table 4 shows that the suggested method succeeded to get the same number of PMUs as obtained by recent studies across all IEEE-bus systems tested. But, the values of SORI are different.

A similar result of comparison may be seen in Table 5 for the second case (with ZIBs). In case of single PMUs failure, the optimal PMUs number, its locations and the value of SORI is given in table 5.

Table 5 Comparison results with available techniques when considering ZIBs

Algorithm	Parameter	IEEE 14-Bus	IEEE 24-Bus	IEEE 30-Bus	NE 39-Bus	IEEE 57-Bus	Algerian network 114-Bus
Proposed Method	N_{PMUs}	3	6	7	8	11	28
	$SORI$	16	29	41	43	60	149
GWO [3]	N_{PMUs}	3	-	7	-	-	35
	$SORI$	15	-	36	-	-	155
MFO [3]	N_{PMUs}	3	-	7	-	-	36
	$SORI$	15	-	36	-	-	146
CS [3]	N_{PMUs}	3	-	7	-	-	34
	$SORI$	15	-	36	-	-	142
WDO [3]	N_{PMUs}	3	-	8	-	-	49
	$SORI$	15	-	37	-	-	190
BGSA [27]	N_{PMUs}	3	-	7	-	-	-
	$SORI$	15	-	36	-	-	-
PSO-GSA [23]	N_{PMUs}	3	-	7	-	11	-
	$SORI$	16	-	41	-	60	-
BPSO [24]	N_{PMUs}	3	6	7	13	-	-
	$SORI$	15	29	36	51	-	-
BPSO [29]	N_{PMUs}	3	6	7	8	11	-
	$SORI$	16	29	41	43	60	-
ILP [30]	N_{PMUs}	3	6	7	8	11	-
	$SORI$	16	27	33	43	60	-
BSDP [12]	N_{PMUs}	3	-	7	-	11	-
	$SORI$	16	-	36	-	57	-
ILP [31]	N_{PMUs}	3	-	7	8	11	-
	$SORI$	16	-	34	43	59	-
BPSO [32]	N_{PMUs}	-	6	7	8	11	-
	$SORI$	-	28	37	40	59	-
ES [33]	N_{PMUs}	3	6	7	8	11	-
	$SORI$	16	27	36	43	60	-

In case of single PMUs failure, the optimal PMUs number, its locations and the value of SORI is given in table 6.

Table 6 Number and position of PMUs in single PMU failure case

Test System	Number of PMUs; N_{PMUs}	Positions of PMUs	SORI
IEEE 14-Bus	7	2, 4, 5, 6, 9, 11, 13	34
IEEE 24-Bus	11	1, 2, 7, 8, 9, 10, 16, 18, 19, 20, 21	46
IEEE 30-Bus	14	2, 3, 4, 7, 10, 12, 13, 15, 16, 18, 20, 24, 27, 30	60
NE 39-Bus	17	2, 6, 8, 10, 13, 16, 18, 20, 21, 23, 25, 26, 29, 34, 36, 37, 38	66
IEEE 57-Bus	23	1, 3, 6, 9, 12, 14, 15, 18, 20, 25, 27, 29, 31, 32, 33, 38, 39, 41, 50, 51, 53, 54, 56	97
Algerian network 114-Bus	59	1, 2, 3, 4, 10, 12, 13, 16, 18, 19, 20, 22, 23, 25, 26, 29, 32, 34, 36, 38, 39, 40, 41, 43, 47, 49, 52, 54, 57, 59, 61, 63, 65, 67, 68, 69, 71, 77, 78, 79, 80, 82, 84, 87, 88, 89, 92, 93, 98, 100, 101, 102, 103, 108, 109, 110, 111, 112, 113	239

In this case, it is evident that the number of PMUs necessary to attain a full Monitoring of the power system will be increased, because each bus is monitored with two PMUs to make sure that the power system stays observable in the event of a single PMUs loss, and the results given in Table 6 agree this idea.

The optimal number of PMUs for Algerian 114-bus network, in case 3 is 59. It is higher than the case 1 and case 2, where the number of PMUs needed is 33 and 28 PMUs, severally.

Table 7 tabulated a comparison results between the suggested technique and some previous studies for the third case (loss of single PMUs).

Table 7 Comparison results with available methods considering single PMU loss

Test System	Proposed Method	PSO-GSA [23]	BILP [38]	GA [39]	ES [33]	HDPSO [40]	ILP [34]
IEEE 14-Bus	7	7	7	7	7	7	7
IEEE 24-Bus	11	-	-	11	13	-	-
IEEE 30-Bus	14	14	15	14	15	16	15
NE 39-Bus	17	-	17	17	18	17	18
IEEE 57-Bus	23	24	25	25	26	23	26
Algerian network 114-Bus	59	-	-	-	-	-	-

For IEEE 30 and bus, 57-bus, the suggested approach managed to get optimal number less than other methods.

Concerning the situation of outage of single line, the places of PMUs have already been identified in a way that outage of any line does not influence on the system monitoring. The optimal PMUs number obtained, the observability and their location by the suggested approach is presented in table 8.

Table 8. Number and position of PMUs in single line outage

Test System	Number of PMUs; N_{PMUs}	Positions of PMUs	SORI
IEEE 14-Bus	7	1, 3, 6, 8, 9, 11, 13	25
IEEE 24-Bus	8	1, 2, 8, 9, 10, 16, 20, 21	38
IEEE 30-Bus	14	2, 3, 5, 6, 10, 11, 12, 13, 15, 17, 19, 24, 26, 29	56
NE 39-Bus	12	2, 3, 6, 8, 13, 16, 20, 22, 23, 25, 26, 29	55
IEEE 57-Bus	21	1, 2, 6, 12, 14, 15, 19, 23, 25, 28, 31, 33, 35, 38, 41, 50, 51, 52, 53, 55, 56	84
Algerian network 114-Bus	49	3, 4, 6, 11, 12, 13, 15, 17, 19, 22, 25, 30, 32, 35, 36, 38, 39, 40, 43, 45, 47, 49, 53, 56, 61, 65, 68, 69, 71, 73, 77, 78, 79, 80, 82, 83, 87, 89, 91, 92, 95, 98, 102, 103, 104, 107, 108, 112, 113	175

The table 9 summarizes the comparative results between the method used and some modern methods for each test system.

Table 9. Comparison results with available methods considering single line outage

Test System	Proposed Method	ILP [37]	BPSO [35]	BPSO [36]	ILP [34]
IEEE 14-Bus	7	7	7	7	7
IEEE 24-Bus	8	-	-	-	-
IEEE 30-Bus	14	14	15	15	15
NE 39-Bus	12	18	17	17	18
IEEE 57-Bus	21	22	22	22	26
Algerian network 114-Bus	49	-	-	-	-

From this table, the optimal PMU numbers are the same in the IEEE 14-bus system. In IEEE 24-bus system the optimal PMUs number is 8 in proposed approach.

In IEEE 30-bus, the numbers of PMUs is lower in the suggested technique compared to [34], [35] and [36] but similar to [37]. In NE 39-bus and IEEE 57-bus, optimum number of PMUs is 12 and 21 respectively in the suggested method, which is less than all the compared methods.

With regard to Algerian 114-bus, the number of PMUs is 49, since there are no other ways to compare with them, the only suggested method that has reached to this solution remains.

Table 10 tabulates comments about several of the solutions proposed in recent studies in case loss of single PMUs.

Table 10 Reported results of articles previously published in case of loss of a single PMU

Methods	Parameter	IEEE 30-Bus	NE 39-Bus	IEEE 57-Bus
ILP [38]	Number of PMUs	15	17	25
	The observability	Unobservable	Unobservable	Observable
	Observations	Loss the PMU at bus 12 leads to unobservable network	Loss of any PMUs leads to unobservable network	The system is observable but with two additional PMUs
BICA [41]	Number of PMUs	13	-	22
	The observability	Unobservable	-	Unobservable
	Observations	Loss the PMU at bus 12 leads to unobservable network	-	Loss of the PMUs at buses 25, 38, 56 leads to unobservable system
ES [33]	Number of PMUs	15	18	26
	The observability	Observable	Unobservable	Observable
	Observations	The system is observable but with one additional PMU	Loss of the PMU at bus 13 leads to unobservable network	The system is observable but with three additional PMUs

For IEEE 30-bus it was mentioned in [38], [41] the number of PMUs is 15 and 13 respectively, however the failure of PMUs at bus 12 for each method causes loss of network observability.

For [33], the system is observable but with one additional PMUs. For NE 39-bus presented in [38], although the number of PMUs is high (five additional PMUs), the loss of any one o them leads to the loss of the network observability.

Table 11 presents comments on some research in case 4 (outage of line).

Table 11. Reported results of articles previously published in case of single line outage

Methods	Parameter	IEEE 30-Bus	NE 39-Bus	IEEE 57-Bus
ILP [37]	Number of PMUs	14	18	22
	The observability	Observable	Observable	Observable
	Observations	The system is observable	The system is observable but with six additional PMUs	The system is observable but with one additional PMU
ILP [42]	Number of PMUs	13	16	19
	The observability	Unobservable	Unobservable	Unobservable
	Observations	Loss one of the branches: 27-28, 29-30; leads to unobservable system	Loss the branch: 10-13; leads to the unobservable system	Loss one of the branches: 22-38, 26-27, 27-28; leads to the unobservable system
BICA [41]	Number of PMUs	11	-	19
	The observability	Unobservable	-	Unobservable
	Observations	Loss one of the branches: 9-11, 12-13, 25-26; leads to the unobservable system	-	Loss one of the branches: 22-38, 26-27, 27-28; leads to unobservable system

On the other side, in IEEE 57-bus, it was found in references [38] and [33] that the system is observable but with 1 and 3 additional PMUs, respectively.

In [41] loss of PMUs at buses 25, 38 and 56 leads to unobservable system. In [37] IEEE 30-bus, NE 39-bus and 57-bus are all observable.

For NE 39 -bus and IEEE 57-bus there are 6 and 1 extra PMUs, Respectively. The results obtained by [42] s shows that the three test systems unobservable in case outage of single line. For example, in IEEE 57-bus loss one of the branches: 22-38, 26-27, 27-28; leads to unobservable system.

For [41], not enough PMUs are available to make the entire system observable. For instance, in case of IEEE 30-bus, failure one of the lines: 9-11, 12-13, 25-26; can lead a loss of network monitoring.

6. Conclusions

In this paper, the OPP problem is solved with the aim of reducing the PMUs number and improving the measurement redundancy, based on SORI values that assess the quality of the placements of PMUs in power system.

This paper deals with four different situations such as ignoring and considering ZIBs, single PMU failure and single line outage. A gravitational search algorithm (GSA) was utilized as an optimization means, which uses the attraction forces between the particles in the universe.

The proposed technique has been tested on six test systems, IEEE 14-bus, 24-bus, 30-bus, NE 39-bus, 57-bus and Algerian 114-bus. For the purpose of verification, the results obtained were compared with other modern methods.

The results obtained indicate that the proposed method succeeded in reducing the number of PMUs and improving the observability of the power system.

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