Measurement of Transformer Core Loss based on Low-Frequency Triangular Wave

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Abstract

For the no-load test of transformer with large capacity and voltage level, it is difficult to manufacture large-capacity variable-frequency sine wave power supply. In order to reduce the capacity and manufacturing difficulty of test power supply, a separation measurement and equivalent method of transformer core loss based on low-frequency triangular wave is proposed. This method is based on Bertotti's core loss separation model, constructs a matrix equation, uses the principle of least squares method to solve the hysteresis loss coefficient, eddy current loss coefficient and excess loss coefficient, and then converts core loss to that under power frequency (50 Hz) sine wave excitation. In addition, in the case of whether excess loss is considered, the analysis and comparison after conversion are carried out respectively. The experimental results show that the core loss based on the proposed method has a good agreement with that under the same frequency actual sine wave excitation. When the voltage amplitude is below 115V, the relative errors of the core losses without considering excess loss are smaller than those converted with excess loss. While the voltage amplitude is greater than 115V, the relative errors of the former are greater than the latter. And in both cases, except for the start of the test, the relative errors between the calculated core losses of triangular wave and the measured sine wave are less than 2.8%. This method has high conversion accuracy and engineering application value.

Keywords: Triangular wave, Transformer, Core loss, Low-frequency method, Least square method.

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1. Introduction

With the increasing of voltage level and transmission capacity of power grid, as the main equipment of power grid, the voltage level and capacity of ferromagnetic components such as transformer, mutual inductor and reactor are also increasing. The performance of these ferromagnetic elements directly affects the safe operation of the power grid [1].

For transformer, the important method to judge its performance is no-load loss measurement test. Through the test, it can be found whether the transformer has defects such as poor insulations between silicon steel sheets, local short-circuits and burns between core poles or chips, and other defects [2-4].

No-load loss of transformer is mainly composed of core loss, copper loss and stray loss [5-6].

Because no-load current and winding DC resistance are small, stray loss and copper loss can be ignored.

Therefore, core loss is the main component of no-load loss of ferromagnetic components. In the no-load test of transformer, the power frequency (50 Hz) rated voltage is generally supplied by one winding, and the other windings are open circuit [7].

However, with the increase of the voltage level and capacity of the transformer, the capacity and voltage requirements of the no-load test power supply are higher, which also causes the safety risks of people and equipment.

Reference [8] mentioned that the core loss is related to the variable frequency of the excitation power supply.

Fiorillo and others proposed an improved approach to power losses in magnetic laminations under non-sinusoidal induction waveform [9].

Roshen presents a very practical, yet very general and accurate model, for core loss calculations in case of non-sinusoidal voltage waveforms [10].

The Chinese national standard [11] proposed a low-frequency measurement method for the excitation characteristics of current transformers, which is very suitable for practical engineering applications.

However, it ignores the voltage on the DC resistance of the winding and increment of eddy current with the increase of frequency, which will lead to a large error between the conversion result and the 50 Hz sine wave excitation measurement method.

CT analyser produced by Austria Omicron company adopts low-frequency power supply for test.

According to the relationship between excitation characteristics and excitation voltage frequency, adjust the frequency of test power supply to ensure that the excitation characteristics of transformer can be accurately measured under the condition of low-frequency and low-voltage.

However, the principle of low-frequency conversion is not disclosed.

Some scholars have used the low-frequency method to measure the core loss of transformer and achieved good results [12-14].

Therefore, a low-frequency measurement and equivalent method for the core loss of a single-phase transformer based on a triangular wave is proposed. A variable frequency triangular wave power supply is used instead of a conventional 50 Hz sine wave power supply. By applying low frequency triangular wave excitation of several frequencies, a matrix equation is constructed based on the Bertotti's core loss separation model.

The hysteresis loss coefficient, eddy current loss coefficient and additional loss coefficient are solved by using the principle of least square method, and then the core loss is converted to that under 50Hz sine wave excitation.

In addition, according to Bertotti's core loss separation model, this paper makes a conversion analysis and comparison on whether excess loss is considered, and judges whether the core loss is in good agreement with that under the 50 Hz sine wave excitation.

2. Measurement method based on low-frequency

2.1 Circuit model of core loss separation for single-phase transformer

The equivalent model of a typical mathematical circuit for the core loss of a single-phase transformer is shown in Figure 1 [15]

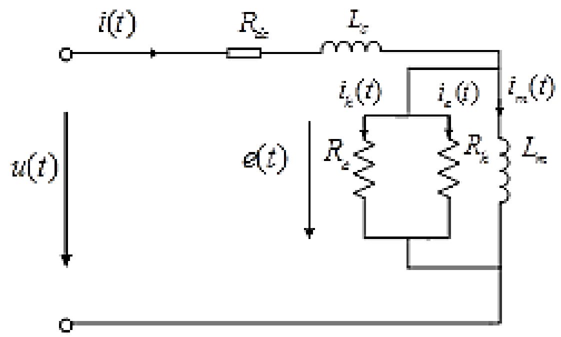

Figure 1. No-load circuit model of single-phase transformer

In figure 1:

- R_e is the equivalent resistance of eddy current loss,
- R_h is the equivalent resistance of hysteresis loss,
- L_m is the magnetizing inductance, and the three constitute the circuit model of main magnetic circuit in parallel,
- R_{dc} is the DC resistance,
- L_σ is the leakage inductance;
- $u(t)$ is the excitation voltage on the test side,
- $e(t)$ is the induced electromotive force of the transformer,
- $i_m(t)$ is the current flowing through the branch where is the magnetizing inductance L_m,
- $i_e(t)$ is the eddy current loss equivalent current,
- $i_h(t)$ is the hysteresis loss equivalent current,
- i(t) is the test excitation current.

It can be seen from Figure 1, that the eddy current loss and hysteresis loss will change with the excitation voltage and the excitation current.

According to formulas (1) and (2) [16], for the same voltage excitation waveform, when the saturation degree of the magnetic circuit (Magnetic flux amplitude Φm) is constant, the induced electromotive force E is directly proportional to the frequency f.

When there is no-load, the voltages of leakage inductance and DC resistance are small, so the excitation voltage U is approximately equal to the electromotive force E.

When the frequency is reduced, the excitation current I will also be reduced, which will reduce the output power of the power supply, thus reducing the capacity of the test power supply [13].

$$e(t) = NA\frac{dB}{dt} \tag{1}$$

$$E = K_v fN\Phi_m \tag{2}$$

In the formulas (1) and (2):

- N is the number of turns of the core,
- A is the cross-sectional area of the core,
- B is the magnetic flux density,
- K_v is the form factor,
- f is the frequency,
- Φ_m is the magnetic flux amplitude.

2.2 The equivalent of magnetic flux generated by triangular wave and sine wave

For sine wave excitation, it is assumed that the induced electromotive force at both ends of the core is:

$$e(t) = E_m \sin(\omega t) \tag{3}$$

where E_m is the peak value of electromotive force.

According to formula (1), the magnetic flux density $B(t)$ is:

$$B(t) = B_m \cos(\omega t) \tag{4}$$

$$B_m = \frac{E_m}{NA\cdot 2\pi f} = \frac{E}{NA\cdot \sqrt{2}\pi f} \tag{5}$$

In the formula (5), B_m is the amplitude of the magnetic flux density, and E is the effective value of the electromotive force.

Similarly, for triangular wave excitation, it is assumed that the induced electromotive force and magnetic flux density at both ends of the core are:

$$e(t) = \begin{cases} E_m(4t/T - 1) & 0 < t < T/2 \\ E_m(3 - 4t/T) & T/2 < t < T \end{cases} \tag{6}$$

$$B(t)=\begin{cases}\frac{E_m}{NA}(2t^2/T-t) & 0<t<T/2\\ \frac{E_m}{NA}(-2t^2/T+3t-T) & T/2<t<T\end{cases} \quad (7)$$

$$B_m=\frac{E_m}{8NAf}=\frac{\sqrt{3}E}{8NAf} \quad (8)$$

It can be seen from equations (5) and (8), that in order to keep the amplitude of the magnetic flux density B_m by the triangular wave and sine wave excitation voltages of the same frequency equal, it is necessary to make:

$$\frac{E_{\sin}}{E_{tri}}=\frac{\sqrt{6}\pi}{8}\approx 0.962 \quad (9)$$

Among them: E_{sin} is the effective value of sine wave electromotive force, and E_{tri} is the effective value of triangular wave electromotive force.

For large-capacity ferromagnetic components, if the voltages on leakage inductance and DC resistance are ignored, the formula (9) can be used to keep the effective value of excitation voltages of sine wave and triangular wave, and the equivalent loss under sine wave excitation can be converted by the volt-ampere characteristic curve and core loss under triangular wave excitation.

2.3 Separation of core loss under the excitation of triangular wave and sine wave

No-load loss of transformer includes core loss (P_c) and copper loss (P_{cu}) caused by no-load current flowing through DC resistance of winding.

According to Bertotti's core loss separation method [6], under sine wave excitation, core loss can be divided into hysteresis loss P_h, eddy current loss P_e and excess loss P_{ex} [5,6,16-17].

$$\begin{aligned}P_c&=P_h+P_e+P_{ex}\\&=k_hB^\alpha f+k_eB^2f^2+k_{ex}B^{1.5}f^{1.5}\\&=W_hf+W_ef^2+W_{ex}f^{1.5}\end{aligned} \quad (10)$$

where:

k_h, k_e, and k_{ex} are the hysteresis, eddy current, and excess coefficients, respectively, which depend on the lamination material, the thickness, and the conductivity among other factors [17].

f is the frequency,

α is the Steinmetz coefficient.

B is the magnetic flux density.

W_h, W_e and W_{ex} are hysteresis loss coefficient, eddy current loss coefficient and excess loss coefficient respectively and are only related to materials B_m at low frequency or 50 HZ.

Therefore, without considering skin effect and excess loss, the calculation formula of the core loss separation under sine wave excitation is as follows [16-18]:

$$\begin{aligned}P_{c-\sin}&=k_hB^\alpha f+k_eB^2f^2\\&=W_hf+W_ef^2\end{aligned} \quad (11)$$

According to formula (10), the hysteresis loss is:

$$P_h=k_hB^\alpha f \quad (12)$$

It is the energy consumed by magnetic materials to overcome the friction of magnetic domain walls during magnetization.

For the same magnetic core, under any excitation waveform, as long as the change amplitude of magnetic flux density and the frequency are the same, the hysteresis loss is the same, that is, the influence of the excitation waveform on the hysteresis loop can be ignored [18-20], and the hysteresis loss is proportional to the frequency.

Therefore, under the same conditions, the hysteresis loss $P_{h\text{-sin}}$ of the sine waveform is equal to $P_{h\text{-tri}}$ of the triangular waveform, that is:

$$P_{h-\sin}=P_{h-tri} \quad (13)$$

In reference [21], the core loss was calculated and derived, and the equivalent formula for eddy current loss was obtained:

$$P_e=\frac{1}{8\pi\rho N^2A}\frac{1}{T}\int_0^T e(t)^2\,dt \quad (14)$$

where:

N is the number of turns,

A is the area of the cross section,

ρ is the resistivity of the core material.

Considering the effective value of electromotive force:

$$E=\sqrt{\frac{1}{T}\int_0^T e(t)^2\,dt} \quad (15)$$

Combining equations (14) and (15), it can be concluded that:

$$P_e=\frac{E^2}{8\pi\rho N^2A} \quad (16)$$

Combining equations (5) and (16), the eddy current loss $P_{e\text{-}sin}$ under sine wave excitation can be obtained as follows:

$$P_{e-\sin}=\frac{\pi A}{4\rho}B_m^2f^2 \quad (17)$$

According to formula (8) and (16), the eddy current loss $P_{e\text{-}tri}$ under triangular wave excitation is as follows:

$$P_{e-tri}=\frac{8A}{3\pi\rho}B_m^2f^2 \quad (18)$$

From equations (17) and (18), it can be seen that the eddy current loss is still proportional to the square of B_m and f, no matter for sine wave or triangular wave excitation, only the difference of proportion coefficient.

If η_e is the conversion coefficient of eddy current loss under sine wave and triangular waveform excitation, then η_e does not change with the degree of saturation of the magnetic circuit and the frequency.

$$\eta_e = \frac{P_{e-\sin}}{P_{e-tri}} = \frac{3\pi^2}{32} \tag{19}$$

In reference [10], the excess loss was calculated and derived, then the equivalent formula for excess loss was obtained:

$$P_{exc} = \sqrt{\frac{A\varepsilon n_0}{\rho}}\left(\frac{dB(t)}{dt}\right)^{\frac{3}{2}} \tag{20}$$

Here, ε is a numerical constant n_0 and characterizes the statistical distribution of the local coercive fields.

If it is assumed a sinusoidal variation in time, then (21) the average excess loss density is given as:

$$P_{ex-\sin} = 3.5\cdot\sqrt{\frac{2\pi A\alpha n_0}{\rho}}(B_m f)^{\frac{3}{2}} \tag{21}$$

Similarly, for the triangular wave voltage waveform, Equation (22) gives the average excess loss density as

$$P_{ex-tri} = 9.05\sqrt{\frac{A\alpha n_0}{\rho}}(B_m f)^{\frac{3}{2}} \tag{22}$$

From formulas (21) and (22), the conversion coefficients λ_{ex} of the excess loss between the sine wave and triangular wave with the same frequency and B_m can be obtained, which is formula (23).

$$\lambda_{ex} = \frac{P_{ex-\sin}}{P_{ex-tri}} = \frac{3.5\sqrt{2\pi}}{9.05} \tag{23}$$

In summary, for sine wave and triangular wave excitation with the same frequency, if their B_m is equal, the hysteresis loss generated by them will be equal, while eddy current loss and excess loss have a fixed multiple relationship. So, there is no fixed proportional relationship, and it will vary with the saturation of the magnetic circuit.

At this point, it is proved that the eddy current loss and hysteresis loss can also be calculated by the core loss separation method under the excitation of triangular wave.

Therefore, when skin effect and excess loss are considered, the calculation formula of modified core loss separation under triangular wave excitation is as follows:

$$\begin{aligned} P_{c-tri} &= k_h B^{\alpha} f + k_e B^2 f^2/\eta_e + k_{ex} B^{1.5} f^{1.5}/\lambda_{ex} \\ &= W_{h-tri} f + W_{e-tri} f^2 + W_{ex-tri} f^{1.5} \end{aligned} \tag{24}$$

In formula (24), $W_{h\text{-}tri}$, $W_{e\text{-}tri}$ and $W_{ex\text{-}tri}$ are the hysteresis loss coefficient, eddy current loss coefficient and excess loss coefficient of triangular wave respectively.

Therefore, when skin effect and excess loss are not considered, the calculation formula of the modified core loss separation under the triangular wave excitation is:

$$\begin{aligned} P_{c-tri} &= k_h B^{\alpha} f + k_e B^2 f^2/\eta_e \\ &= W_{h-tri} f + W_{e-tri} f^2 \end{aligned} \tag{25}$$

If it is guaranteed that the B_m generated by triangular waves of different frequencies are equal, $W_{h\text{-}tri}$, and $W_{e\text{-}tri}$ can be considered as constants.

Therefore, the values of $W_{h\text{-}tri}$, and $W_{e\text{-}tri}$ can be calculated by the core loss generated by several different frequencies triangular wave excitations respectively, and then the core loss under the same frequency or even 50 Hz sine wave excitations can be calculated.

3. Equivalent calculation method

3.1 Equivalent calculation of core loss separation considering excess loss

Under the premise of ensuring that the B_m generated by the triangular wave of each frequency is equal, apply m ($2 \le m \le 4$) frequencies (f_1, f_2,..., f_m) low-frequency triangular waves voltages to the core until it is saturated. The high-speed measuring device records each excitation voltage $u(t)_{stri_i}$ and current data $i(t)_{tri_i}$ (i=1,2,...,m).

Calculate respectively: the effective values of the excitation voltage U_{tri} and current I_{tri} of m triangular waves are applied; and use formulas (26) to (27) to calculate the induced electromotive force $e_{tri}(t)$ and its effective value E_{tri} and core loss $P_{c\text{-}tri}$.

Since the conversion steps are the same for each frequency, for the sake of brevity in formula expression, the calculated intermediate quantity does not have a subscript i.

$$e_{tri}(t) = u_{tri}(t) - i_{tri}(t)R_{dc} - L_{\sigma}\frac{di_{tri}(t)}{dt} \tag{26}$$

$$P_{c-tri} = \frac{1}{T}\int_0^T u_{tri}(t)i_{tri}(t)dt - I_{tri}^2 R_{dc} \tag{27}$$

Take the core loss data of m frequencies into equation (24) to get:

$$\begin{cases} P_{c-tri_1} = W_{h-tri} f_1 + W_{e-tri} f_1^2 + W_{ex-tri} f_1^{1.5} \\ P_{c-tri_2} = W_{h-tri} f_2 + W_{e-tri} f_2^2 + W_{ex-tri} f_2^{1.5} \\ \qquad \cdots \\ P_{c-tri_m} = W_{h-tri} f_m + W_{e-tri} f_m^2 + W_{ex-tri} f_m^{1.5} \end{cases} \tag{28}$$

Equation (28) can be expressed in matrix form as follows:

$$FW = P \tag{29}$$

where:

$$F=\begin{bmatrix} f_1 & f_1^2 \\ f_2 & f_2^2 \\ \cdots & \cdots \\ f_m & f_m^2 \end{bmatrix},\ W=\begin{bmatrix} W_{h-tri} \\ W_{e-tri} \\ W_{ex-tri} \end{bmatrix},\ P=\begin{bmatrix} P_{c-tri_1} \\ P_{c-tri_2} \\ \cdots \\ P_{c-tri_m} \end{bmatrix}$$

where:

F is the frequency matrix,

W is the vector composed of the parameters W_{h-tri}, W_{e-tri} and W_{ex-tri},

P is the core loss vector.

For overdetermined linear equations (28), the least square method is used to solve W_{h-tri}, W_{e-tri} and W_{ex-tri}:

$$W=\left(F^T F\right)^{-1} F^T P \tag{30}$$

Therefore, the core loss P_{c-sin} converted to sine wave of the same frequency by inverse derivation is as follows:

$$\begin{aligned} P_{c-\sin} &= W_{h-\sin} f + W_{e-\sin} f^2 + W_{ex-\sin} f^{1.5} \\ &= W_{h-tri} f + \eta_e W_{e-tri} f^2 + \lambda_{ex} W_{ex-tri} f^{1.5} \end{aligned} \tag{31}$$

In the formula (31): W_{h-sin}, W_{e-sin} and W_{ex-sin} are the coefficient of hysteresis loss, eddy current loss and excess loss converted to sine wave of the same frequency, respectively.

Similarly, the core loss P_{cn} converted from a triangular wave excitation to a 50 Hz sine wave excitation is:

$$P_{cn} = W_{h-tri} f_n + \eta_e W_{e-tri} f_n^2 + \lambda_{ex} W_{ex-tri} f_n^{1.5} \tag{32}$$

In the formula (32): f_n is the 50 Hz.

3.2 Equivalent calculation of core loss separation without considering excess loss

There is the same as Section 2.1, when skin effect and excess loss are not considered, and the B_m generated by triangular waves of each frequency are equal, apply m (2≤m≤4) frequencies (f_1, f_2,…, f_m) low-frequency triangular waves voltages to the core until it is saturated.

Then the separation coefficient equations of core loss are:

$$\begin{cases} P_{c-tri_1} = W_{h-tri} f_1 + W_{e-tri} f_1^2 \\ P_{c-tri_2} = W_{h-tri} f_2 + W_{e-tri} f_2^2 \\ \qquad \cdots \\ P_{c-tri_m} = W_{h-tri} f_m + W_{e-tri} f_m^2 \end{cases} \tag{33}$$

Similarly, W_{h-tri} and W_{e-tri} can be obtained by using the least square method, and the core loss P_{c-sin} converted to sine wave excitation of the same frequency is:

$$\begin{aligned} P_{c-\sin} &= W_{h-\sin} f + W_{e-\sin} f^2 \\ &= W_{h-tri} f + \eta_e W_{e-tri} f^2 \end{aligned} \tag{34}$$

Similarly, the core loss P_{c-sin} converted to 50 Hz sine wave is P_{cn}:

$$P_{cn} = W_{h-tri} f_n + \eta_e W_{e-tri} f_n^2 \tag{35}$$

4. Experiment and analysis

4.1 Experiment

A variable-frequency power supply, a high-speed measuring device and a single-phase transformer are used as the experiment device, as shown in Figure 2.

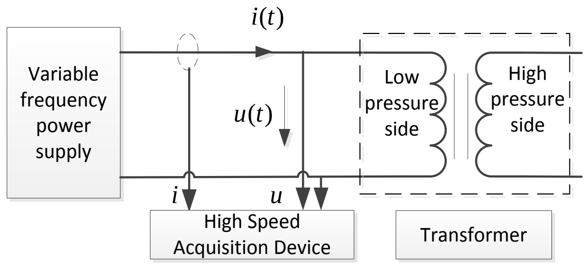

Figure 2. The schematic diagram of the experimental device

Among them, the variable-frequency power supply is PCR2000LE, which can produce triangular and sine waves with variable frequency (0.1 Hz ~ 999.9 Hz). The high-speed measuring device uses the Teledyne LeCroy Oscilloscope HDO8000 to record the voltage and current data at both ends of the winding (Voltage probe: HVD3106, current probe: CP030A).

The tested object is a single-phase transformer, and the parameters at the low-voltage side of the transformer are listed in Table 1.

Table 1. The parameters of tested transformer in low-voltage side

Parameters	Value
Nominal Capacity/kVA	16.7
Voltage Ratio	380V/220V
No-load Current	<1.5%
No-load loss	<1.2%
Impedance Voltage	14%
DC Resistance /Ω	1.807×10^{-2}
Leakage Inductance/H	6.432×10^{-4}

According to Figure 2, the high voltage side winding of the transformer is open, and voltage is applied from the low voltage side.

Apply sine wave and triangular wave of 3 frequencies (10 Hz, 15 Hz, 20 Hz) to the transformer to perform no-load test and ensure that the magnetic flux amplitudes at each frequency voltage are correspondingly equal.

For the triangular wave voltage with frequency f, the effective range of applied voltage is: 0~[f/(0.962fn)]•1.2Un (where: Un is the effective value of the rated voltage on the voltage side, "1.2" represents 1.2 times of the rated voltage normally applied during excitation characteristic test in Engineering).

4.2 Results and analysis

In order to verify whether the generated flux amplitudes Φ_m by triangular wave excitation of different frequencies are equal, the flux amplitudes Φ_m of different frequencies are calculated in the experiment, and the changes of with excitation voltages are calculated.

Through the comparative analysis of the experimental data, it can be seen that in the process of the test, when three triangular waves of frequency (10Hz, 15Hz, 20Hz) are applied, the generated flux amplitudes Φm by the triangular waves of each frequency are basically the same under the same voltage.

Moreover, the generated flux amplitudes Φm under the excitation of 50Hz sine wave are basically the same with those of the three triangular waves.

In the experiment, the measured values of core loss of triangular wave and those of corresponding frequency sine wave are shown in Table 2.

Table 2. The core losses of triangular waves and sine waves at corresponding frequencies

Magnetic flux amplitude Φ_m	The core loss of sine wave			The core loss of triangular wave		
/Wb	10Hz	15Hz	20Hz	10Hz	15Hz	20Hz
0.09	0.162	0.239	0.350	0.191	0.249	0.355
0.18	0.533	0.800	1.173	0.502	0.826	1.220
0.27	1.044	1.640	2.403	0.997	1.683	2.497
0.36	1.682	2.749	4.010	1.683	2.822	4.141
0.45	2.447	4.077	5.987	2.468	4.194	6.169
0.54	3.360	5.634	8.308	3.410	5.795	8.567
0.63	4.409	7.454	10.971	4.496	7.654	11.284
0.68	4.989	8.465	12.426	5.110	8.666	12.769
0.72	5.608	9.514	14.011	5.743	9.767	14.332
0.77	6.260	10.647	15.628	6.405	10.893	16.002
0.81	6.966	11.831	17.405	7.164	12.105	17.806
0.86	7.749	13.149	19.290	7.934	13.420	19.755
0.90	8.563	14.498	21.321	8.778	14.844	21.804
0.95	9.464	16.009	23.475	9.683	16.315	23.999
0.99	10.391	17.532	25.728	10.668	17.938	26.311
1.04	11.401	19.268	28.206	11.753	19.726	28.873
1.08	12.521	21.184	30.976	12.957	21.616	31.677

It can be seen that the triangular waves and sine waves of the same frequencies are excited, and under the condition that Φm is equal, the core losses of the triangular waves are almost greater than those of the sine waves.

4.2.1 Analysis of core loss separation considering excess loss

In order to better analyse the differences of core loss under triangular wave and sine wave excitation, the core loss is decomposed into three components: hysteresis loss, eddy current loss and excess loss.

Taking 20 Hz triangular wave and sine wave as examples, as shown in formula (10), the hysteresis loss, eddy current loss and excess loss of the core losses can be obtained as shown in Figure 3.

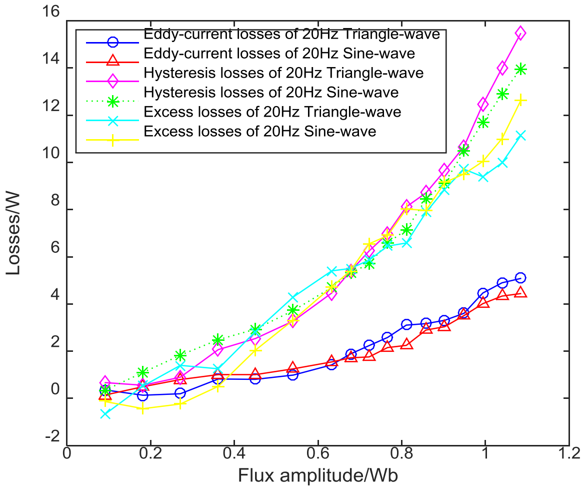

Figure 3. Comparisons of the core loss components of the same frequency sine wave and triangular wave considering excess loss

It can be seen from Figure 3 that the hysteresis losses of the sine wave and triangular wave of the same frequency are closer to equal than the eddy current loss and excess loss.

It shows that when the magnetic flux amplitudes are between 0.15 W_b and 0.68 W_b, the hysteresis losses of the triangular wave are smaller than those of the sine wave, while at other values, the former are greater than the latter. In the whole range of magnetic flux amplitude variation, considering the allowable range of experimental errors, the hysteresis losses of the sine wave and the triangular wave are nearly equal.

Similarly, it shows that the flux amplitude is between 0.15 W_b and 0.68 W_b, the eddy current losses of triangular wave are smaller than those of sine wave, while the former are larger than the latter at other values. At 0.95 W_b, the losses of the two are almost equal.

From the Figure 3, it can be seen that the excess losses of triangular wave are greater than those of sine wave when the flux amplitude is between 0.15 W_b and 0.68 *Wb*, and the excess losses of both waves are cross changing in the whole flux amplitude range.

According to formula (31), keep the hysteresis loss of triangular wave unchanged, multiply eddy current loss and excess loss by loss conversion coefficient respectively, and then get the conversion values from the core loss of transformer excited by triangular wave to sine wave of the same frequency.

The comparisons after conversion are shown in Figure 4.

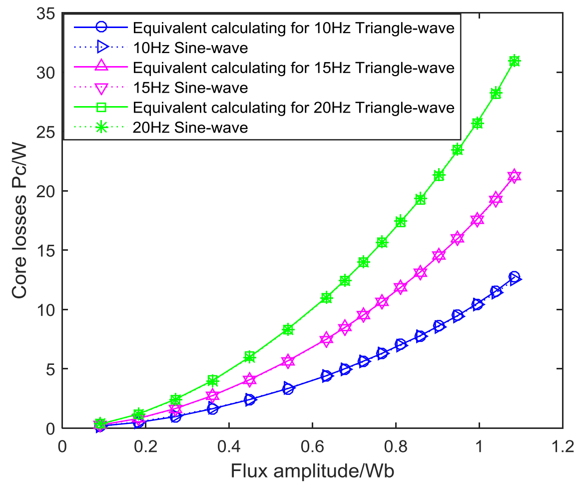

Figure 4. Comparisons of the core losses of sine wave and the converted that of triangular wave at the same frequency considering excess loss

As can be seen from Figure 4, the converted core losses almost overlap with that of the measured sine wave.

4.2.2 Analysis of core loss separation without considering excess loss

Similarly, taking 20 Hz triangular wave and sine wave as examples, as shown in formula (11), the core loss is decomposed into hysteresis loss and eddy current loss, as shown in Figure 5.

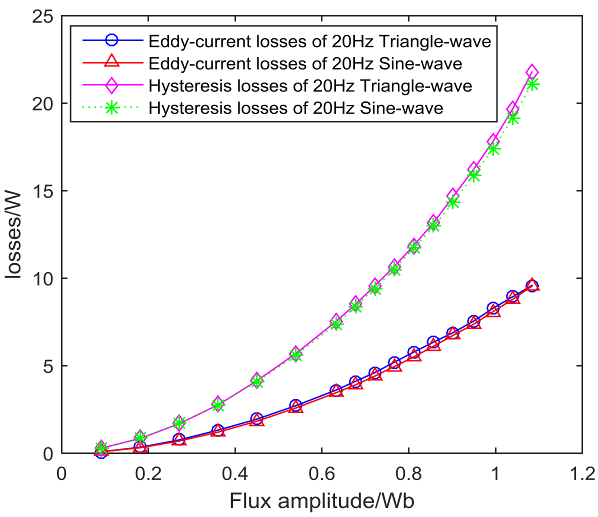

Figure 5. Comparisons of the core losses components of sine wave and triangular wave at the same frequency without considering excess loss

It can be seen from Figure 5 that the hysteresis losses and eddy current losses of the triangular wave and the sine wave of the same frequency are almost same, and the former are slightly larger than the latter.

According to formula (34), keep the hysteresis loss of triangular wave unchanged, multiply the eddy current loss by the loss conversion coefficient to obtain core loss conversion value of the sine wave at the same frequency.

The comparisons after conversion are shown in Figure 6.

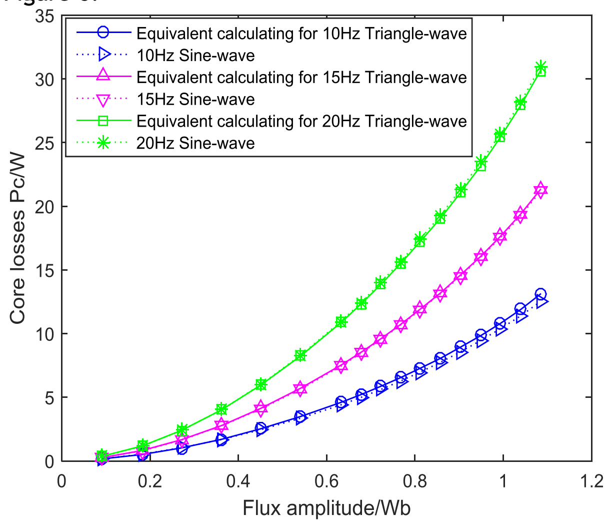

Figure 6. Comparisons of the losses of sine wave and the converted core losses of triangular wave at the same frequency without considering excess loss

It can be seen that the converted core losses are almost the same as those of the measured sine wave, and the converted core losses of 10 Hz triangular wave vary greatly with the increase of magnetic flux amplitude, and these are larger conversion deviation than those of the other two frequencies.

4.2.3 Analysis and comparison

In order to further analyse the core loss separation model, whether or not the residual loss is taken into account, the conversion error of core loss of which model is smaller.

Use formulas (32) and (35) to convert the core losses of 10, 15, 20 Hz triangular wave to those of the 50 Hz sine wave, as shown in Figure 7.

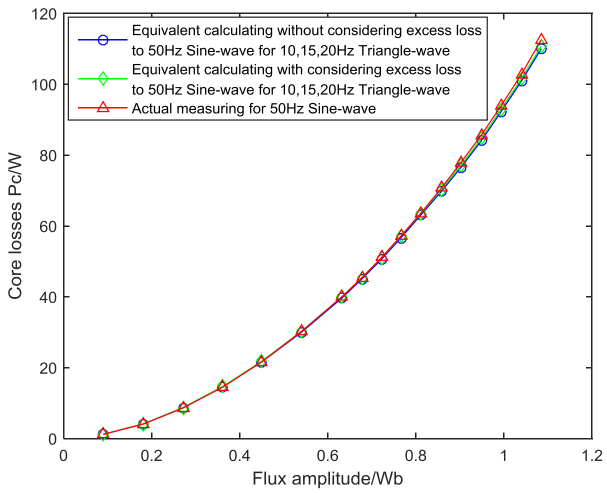

Figure 7. Comparisons of the converted core losses of triangular wave from 10,15,20 Hz to 50 Hz and those of the measured sine wave at 50 Hz considering excess loss and without considering excess loss

It can be seen from Figure 7, that the core losses based on the two models are converted to those of the 50 Hz sine wave, which have a good agreement with the measured core loss under a 50Hz sine wave excitation.

The converted deviations of core losses of 10, 15, 20 Hz triangular wave without considering the excess loss become larger as the magnetic flux amplitude increases than those considering the excess loss under the same conditions.

In order to quantitatively evaluate the coincidence effect between the core losses based on two models and those by being measured under the excitation of 50 Hz sine wave, the relative error is introduced as an evaluation index and calculated, as shown in Figure 8.

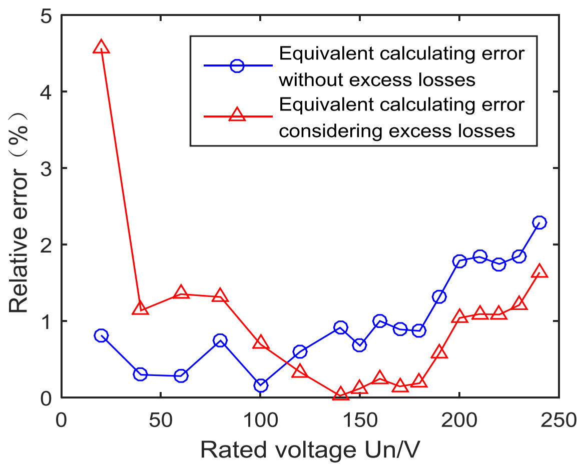

Figure 8. Comparisons of the relative error curves of the converted core losses for triangular wave from 10,15,20 Hz to 50 Hz considering excess loss and without considering excess loss

From Figure 8, it can be seen that, when the voltage amplitude is below 115 V, and the relative errors of core losses after the conversion without considering excess loss are less than that considering excess loss, which indicate that the former are better than the latter in coincidence with the measured core losses under the 50 Hz sine wave excitation, while the effect is opposite when the voltage amplitude is greater than 115 V, the latter are smaller in relative error and better in the coincidence degree than the former.

Except for the first test data, the relative errors of the latter are 4.5 %, and the relative errors of both are less than 2.28 % for other voltage amplitudes.

Therefore, considering the experimental errors, under the triangular wave excitation, the two core loss separation models mentioned in the article are used, and the converted core losses of the transformer have a good conversion effect.

5. Conclusions

Using Bertotti's core loss separation model of transformer, based on several low-frequency triangular wave excitations, the corresponding algorithm is proposed to convert the core losses to sine wave at the same frequency and 50 Hz in the case of considering excess loss and without considering excess loss, respectively.

Compared with the measured core loss of actual sine wave at the same frequency, it has a good agreement. Moreover, when the voltage amplitude is below 115 V, the relative errors of the converted core losses without considering the excess loss are less than those of considering excess loss; when the voltage amplitude is greater than 115 V, the relative errors of the two models are opposite.

Within the allowable range of experimental error, the two core loss separation models proposed in the article can be used to convert core loss of the transformer under triangular wave excitation replacing sine wave excitation.

This method is an extension of the existing low-frequency test method. It inherits the advantages of low-frequency method that can reduce the test power capacity, improve the safety and convenience of the experiment, and at the same time, the manufacturing difficulty and cost of the variable-frequency non-sinusoidal power supply are much smaller than those of the same sine wave.

Therefore, this method can reduce the test cost and has engineering practical value. It provides experimental cases for transformer core loss under non-sinusoidal excitation.

The next step is to analyse and compare the no-load characteristics under the excitation of triangular wave and sine wave based on the proposed two models.

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