Obtaining Probabilistic Characteristics of Electrical Quantities and their Imbalances

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Abstract

The electric power system elements are designed to transmit electricity to consumers. To ensure the transmission, in the design and construction of the power system provide for increased cross-section of lines, throughputs of transformers, cost for other procedures. To find the magnitude of the increase, it is necessary to determine the maximum power flow through the element. This is important for the digital substation operation, where all actions are subordinated to the automation elements, and the ability to control power flows, through the electrical equipment management, has the primary task. This solution is not absolutely effective, since it's not known how much the measures are related to the accident probability. It's needed to know where the maximum power consumption can appear. Therefore, it is necessary to determine in a non-deterministic form how the available power plants power is distributed over the grid to ensure power supply. The main problem is that obtaining the probabilistic characteristics of the parameters by standard statistical methods has no practical solution. In this regard, the method of selection of interval boundaries of input and output data (SIBD) is proposed, and its application for obtaining probabilistic characteristics of electrical quantities and their imbalances.

Keywords: Probability density, Random variable, Quantity, Imbalance, Electric power system.

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1. Introduction

Obtaining the probability distribution function (PDF) and probability density (PD) of multidimensional functional dependence (FD), as well as probabilities of a FD assuming values from the multidimensional domain, in its canonical analytical variant represents a functional of a multidimensional PD within the limits determined by the essence of the problem. In this case, a multidimensional PD is implemented as a product of one-dimensional PDs of continuous random variables (RV), and the functional's limits are revealed by analysing the boundaries of all the RVs determined by the problem's content, starting from the simplest and most specific limits (possibly numerical values) of the outer one-dimensional integral using one selected RV, and proceeding deeper into inner integrals to determine analytical expressions of increasingly complex and multidimensional dependencies of the limits.

There is the practical problem of the mass peculiarities of FDs that shape the boundaries of the area of the functional [1], which condition the immeasurable analytical complications of controlling the expressions of limits and the correct sequence of the algorithm for calculating the functional of the problem. In the electric power industry, first of all, these are the multidimensional problems of determining the probabilistic characteristics of the steady-state mode parameters, electrical quantities during breakdowns, indicators of functional reliability, technical, resource, and economical efficiency, etc. In the case of digital substations (DS), where a high level of automation [2], information exchange between DS elements through various protocols [3], the ability to predetermine the imbalances and manage them will affect the reliability and performance.

However, analytical difficulties of forming limits and analytical calculation of functionals may arise starting from the third or the fourth FD dimension. This is why it becomes pertinent to search for other methods of calculation that are less dependent on FD dimension.

2. SIBD Method Logic

A method of selection of interval boundaries of input and output data (SIBD) [4] that is not critical to FD dimension was proposed. This method relies on the assumption that if all RVs of an FD are equally probable, then the FD value has the same probability as the RVs. The algorithm of SIBD implementation presented in [5] is based on the PDF and PD of random variables and their functional dependence, and ensures the unambiguity of probabilistic values of RVs and FD (due to the binding of random variable values to non-decreasing probability distribution eigenfunctions) and the equal probability of assuming values from the intervals linked with the left boundaries of the PDF of the random variables and the FD, which occurs due to the averaged values of probability eigen-densities in these intervals.

However, the generally correct required logic of the SIBD method relative to equal probabilistic values of RV and FD and the attempt to ensure this equality by means of joint use of eigen PDFs and PDs of every RV and FD as a sufficient condition does not eliminate the loss of a number of fundamental properties of the algorithm of canonical analytic transformation of multidimensional functionals that allow to determine the LPs of multidimensional FDs, including those simulating the problems of the electric power industry in question.

This is demonstrated by validating the SIBD method [4] by comparing the curves of a true PDF (i.e. determined by the functional) and a PDF determined using the proposed SIBD method. It can be noted that for additive (mode parameters and electrical quantities during breakdowns) and multiplicative FDs, the mismatches of the curves of the true PDFs and those obtained using SIBD are significantly different; moreover, in the case with additive FDs, the mismatch increases in the range of the first half of the FD values with the increase of the number of addends.

However, the desired experience of the proposed method does not vanish completely and can be applied in a new refined way, its description is presented below.

The method of determining PDFs and PDs of a FD, as well as that of the probability of an FD assuming a value from the interval of the PDF and PD of random variables, consists in natural summation of probabilities of each variant of RV values combination according to different criteria: the natural criterion of non-exceedance of a certain specified value by FD values is used for PDFs, while for PDs the rational criterion is that of equality of the FD to this specified value. The specified value of FD can be set in arbitrary way for every system of RV variant. However, there are many specified values of an FD in its range. But since the PDF and PD of a functional dependence is formed through combining RVs, the number of specified values increases immeasurably and choosing a variant in this kind of manifold becomes problematic.

A degenerate transformation of this manifold becomes preferable, which will decrease the number of variants. Naturally, such unambiguous transformation is a PDF of a random value [6, 7]. A PDF ensures, in probability measure, the equal range of values of each random value in both one- and multidimensional cases. That said, due to PDF being non-decreasing, its rigid unambiguity is ensured with all values of the random value. This unambiguity is obvious and simple in one-dimensional cases. In multidimensional RV compositions, the functional dependence as a transformation ensures the degeneration of RV multidimensionality down to one-dimensional FD values and, consequently, to the unambiguity and simplicity of a one-dimensional PDF. Each FD value is formed from a manifold of variants of combining all RVs to each FD value. The manifold of RV value variants that corresponds to the FD value can be called an association of this value of the FD. Value-association of the FD that as an event consists of the total sum of variants of combining value-events of all RVs. Variants of combinations of these value-events are incompatible, because they are formed from all RVs and can not combine other than as different values of at least one RV. Values of each RV are incompatible relative to each other, so all variants of combining all RVs will be incompatible as well. Therefore, the probability of association-value of an FD equals the sum of probabilities of all RV combination-events that are associated with this value of the FD, and this probability can be called the eigen-probability of the FD value [8].

The number of combination-events of the RVs of every association is different and depends on the RV number and the FD value in this association's range. In this case, it is necessary to select and unambiguously designate the variant of combining each associations' RVs, thus unambiguously determining the base of the association or the combination-event of the RV, relative to which in can be formed. Such variant of combining all RVs for each association was found, it is the combination of quantiles of one order of all RVs of each association or each value of a functional dependence. Changing the quantile order from zero to one covers the range of every RV and, consequently, the range of combination of all RVs, which means that the FD range is covered too. However, with each single order of quantiles of all RVs, it is possible to account for the probability of only one combination of RV values, which is included in the probabilistic characteristic of the FD association. The entire composition of each FD value represents the sum of probabilities of the entire RV system and the criterion of selecting their combinations, under which one and the same correlation (value) of this criterion is ensured. The criteria for combining components of associations of different probabilistic characteristics of FDs are different: PDF values of each association are determined by FD values that do not exceed the FD from combining all RVs of one order, while PD values for a similar association are determined by one and the same value of the FD of combining all RVs of the same order. That said, the combinations of all RVs for both PDFs and PDs can be quantiles of any different orders as long as FDs of RV values are equal to the FDs of quantiles of RVs of one order. These criteria were set on the basis of the properties of the PDF and PD of a functional dependence [9], RVs, and joint probabilistic characteristics of RV distribution, and unambiguously rigidly order the FD scale for both PDFs and PDs.

This way, the sum of probabilities of the combinations of an association's RV values is a probability of the value of FD as a one-dimensional random value. Components of the proper sum are formed as the probability product of all RVs values of the functional dependence. Probabilities are specified in the product of probabilities of independent RV values, and in the case with the dependent ones so are the conditional probabilities, which have other values depending on the conditions, in which they are considered. In the FDs that describe practical problems, it is rational to consider all RVs as independent variables and, consequently, to change their values arbitrarily, irrespectively of the types of conditional probabilistic characteristics that change with the change of RV values, which is a natural advantage. In the case with dependent RVs, values of one RV must be determined using expressions through products of conditional probabilities, which represents additional counterproductive procedures that can always be avoided using high-quality logical selection of independent RVs or high-quality orthogonalization of the source dependent RVs. This way, it can be assumed that the RVs of all FDs that describe applied problems are independent.

3. SIBD Algorithm for obtaining multidimensional PD

The SIBD algorithm (Figure 1) is described by mathematical operations for explicit expressions for the PD and implemented programmatically according to the capabilities of the software tool for FD obtaining, as a sum of 2 RV, 3 RV, 4 RV, etc.

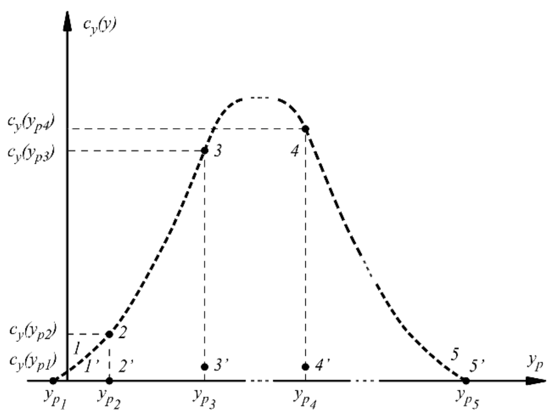

Figure 1. PDs of the sum of two independent uniform RVs

The PDFs for $X_1,\ldots X_2,\ldots X_n$ RV are assumed to be known.

1. Selection of the quantity and the values of the PDF quantile orders: 0, p_1, ... p_j,...1.
2. Forming the PD values as quantiles of the same order.

For example, quantile 0 is expressed by the formula (1):

$$\begin{cases} x_{10} = \mathrm{f}_1^{-1}(0), \ldots, x_{i0} = \mathrm{f}_i^{-1}(0), \ldots, x_{n0} = \mathrm{f}_n^{-1}(0) \\ c_1(x_{10}), \ldots, c_i(x_{i0}), \ldots, c_n(x_{n0}) \end{cases} \quad (1)$$

Similarly, for other quantile orders $p_1,\ldots p_j,\ldots 1$.

3. Forming the FD main orders and compatible PD of the same order.

a) 0 order (2):

$$\begin{cases} y_0 = \varphi(x_{10}, \ldots x_{i0}, \ldots x_{n0}) \\ c_0(x_{10}, \ldots x_{i0}, \ldots x_{n0}) = c_1(x_{10}) \ldots c_i(x_{i0}) \ldots c_n(x_{n0}) \end{cases} \quad (2)$$

b) p_1 order (3):

$$\begin{cases} y_{p_1} = \varphi(x_{1p_1}, \ldots x_{ip_1}, \ldots x_{np_1}) \\ c_{p_1}(x_{1p_1}, \ldots x_{ip_1}, \ldots x_{np_1}) = c_1(x_{1p_1}) \ldots c_i(x_{ip_1}) \ldots c_n(x_{np_1}) \end{cases} \quad (3)$$

Similarly, for other quantile orders ...p_j,...1.

4. Forming and exhaustive search the RV values as quantiles of the selected orders $X_1,\ldots X_2,\ldots X_n$ of FD $Y = \varphi(X_1,\ldots X_2,\ldots X_n)$, calculation of the FD values, compatible PD of RV, it selection and summing as a PD components according to the principle of equality to main orders FD.

4. Full Probabilistic Characteristics for Sums of Independent Random Variables

Based on the described algorithm in Matlab the program was compiled and calculated some PDs of the two- and three-dimensional, and other small dimensions of the FDs, the nature of which is similar to the used values of the electric power industry reliability problems.

4.1. The sum of two independent uniform RV

Two laws of uniform density are given in the same interval (0.1), the expression (4) is used:

$$\begin{cases} \mathrm{f}_1(x) = 0 < x < 1 \\ \mathrm{f}_1(y) = 0 < y < 1 \end{cases} \quad (4)$$

To obtain the density integration (5) is performed:

$$\begin{cases} z < 0, G(z) = 0 \\ 0 < x < 1, G(z) = \frac{z^2}{2} \\ 1 < x < 2, G(z) = 1 - \frac{(2-z)^2}{2} \\ z > 1, G(z) = 2 \end{cases} \quad (5)$$

The result will be a Simpson's law [10, 11] distribution.

This example has no semantic meaning but is set to determine the approximate number of calculations for the two RV FD [11, 12]. However, using the SIBD method described earlier, it is possible to obtain the PD without transforming the integration element and differentiating the expression.

The result is shown in Figure 2.

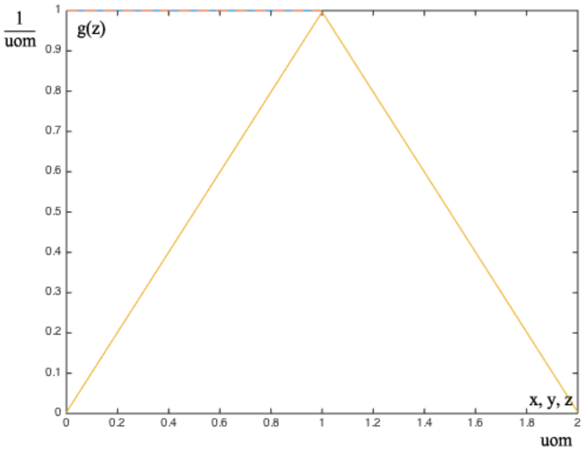

Figure 2. PDs of the sum of two independent uniform RVs

4.3. The sum of two independent exponential RV

For two independent exponential RVs defined on an interval (0, ∞) with a parameter (6), it is easy to find the equation *Z* = *φ(X+Y)* PD [2]. Since variables *X* and *Y* are independent, then f(*x*,*y*) = $f_1(x)f_2(y)$.

$$f_1(x) = f_2(y) = \begin{cases} \lambda e^{-\lambda x}, if\ x \geq 0 \\ 0, \text{otherwise} \end{cases} \quad (6)$$

Based on this, differentiating the expression (6) by variables x and y will get the PD equation (7).

$$\begin{cases} z > 0, \int_0^z \lambda e^{-\lambda(z-y)} \lambda e^{-\lambda y} dy = \int_0^z \lambda^2 e^{-\lambda z} dy = \lambda^2 z e^{-\lambda z} \\ z < 0, g(z) = 0 \end{cases} \quad (7)$$

The system of equations (8) is obtained:

$$g(z) = \begin{cases} \lambda^2 z e^{-\lambda z}, if\ z \geq 0 \\ 0, \text{otherwise} \end{cases} \quad 8)$$

The result is presented in Figure 3.

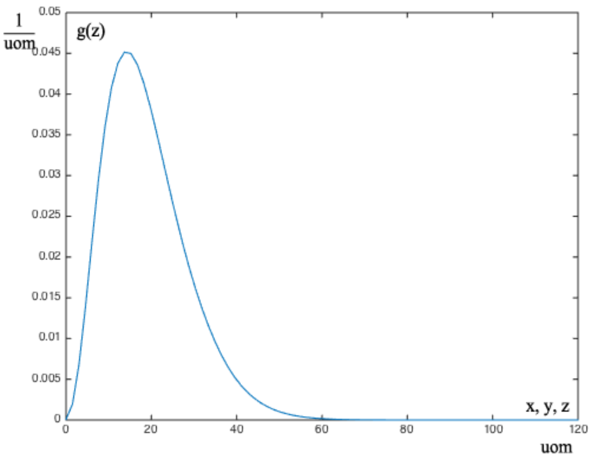

Figure 3. PDs of the sum of two independent exponential RV

4.3. The sum of uniform and normal RVs

In the case of normal and uniform RVs (9) [13]:

$$\begin{cases} f_1(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-m)^2}{2\sigma^2}} \\ f_2(y) = \frac{1}{\beta-\alpha} \end{cases} \quad (9)$$

for *α* < y < *β*.

Using the convergence formula, we have:

$$G(z) = \frac{1}{\beta-\alpha}\int_\alpha^\beta \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-y-m)^2}{2\sigma^2}} dy = \frac{1}{\beta-\alpha}\int_\alpha^\beta \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-(z-m))^2}{2\sigma^2}} dy \quad (10)$$

The integrand in the expression is the RV probability that, subject to this law, falls into the section from α to β, therefore:

$$g(z) = \frac{1}{\beta-\alpha}\left[\Phi^*\left(\frac{\beta-(z-m)}{\sigma}\right) - \Phi^*\left(\frac{\alpha-(z-m)}{\sigma}\right)\right] \quad (11)$$

The obtained PD with parameters *α* = -2, *β* = 2, *m* = 0, *σ* = 1 is presented below (Figure 4).

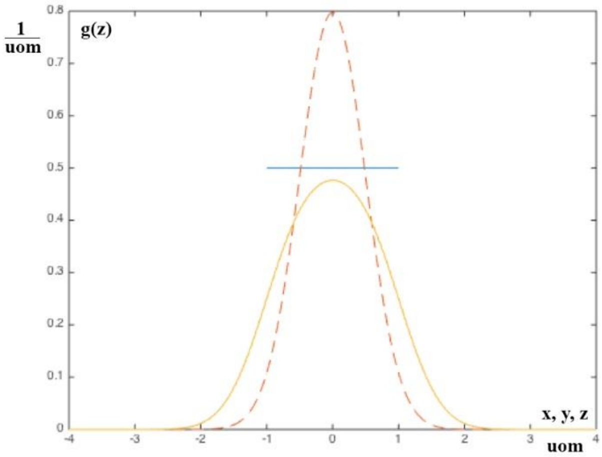

Figure 4. PDs of the sum of two independent uniform RV

5. Available power distribution over the grid

The available power of each power plant provides the required load of consumers and some reserve of the power plant itself, which covers the actual power required from the power plant and remains some reserve for unforeseen needs for emergency and scheduled repairs of equipment. However, in the grid of both the power plant itself and the adjacent areas of the system, the concept of available power is usually not considered as having no physical relation to the power plant.

In fact, it is not. Each grid element is designed to perform the function of passing the actual electrical power to consumers from power plants. In an emergency, the grid elements must pass to the consumer all the necessary power. In an accident, the elements must transmit all the required power.

To ensure the transmission, in the design and construction of the power system provide for increased cross-section of lines, throughputs of transformers, cost for other procedures [14]. To find the magnitude of it increase determined the maximum power flow. But what value should be provided for this increase, since power flows under operational conditions change randomly. The logical and legitimate answer is to find the maximum limit value of the power flows. This decision could be final, but ineffective, because maximum limit value may be rare [15].

Therefore, in order to prevent an increase, the maximum limit, funds will be required that are used to amplify the electrical circuit. It is also necessary to know where the increased power can appear as not from power plants, i.e. due to the reserve provided in the composition of the available power plants power and the possibility of the grid to pass power to the consumer through the grid. But this is a random process, determined by the need for increased power consumption at a particular place in the grid a limited amount of available power at each power plant, and the technical ability to deliver the required power.

It is necessary to develop scenarios to determine how the available power plants power is distributed across the grid to provide power to consumers, not in the deterministic form, which are random, but in the PD form, i.e. flows of active and reactive power in the branches, magnitudes and angles of stress on the tires of power plants and substations in nodes. The received PD can be used to calculate the risks of overload due the operating conditions of each grid resource, as well as due to a decrease in the expected flow of available power along the branch due to accidental damage to power units and grid elements ensuring production and delivery of generating power to the consumer [16].

The analysis showed that two models can be proposed for the processes of production and distribution of available power for consumers. The first is based on the available power plants power of for grid elements, and the second on physical representations, similar to the actual power flows in the grid components according to the electrical engineering laws. According to the first model, the flows distribution in the elements should be in accordance with the engineering data and the actual power flows among the elements, but to some extent arbitrary, in particular, the PD and PDF in each branch can take its own, possibly similar to the available power of the nearest power plant. The second model involves the available power distribution over the grid in accordance to the electrical engineering laws, taking into account to the operation rules and characteristics of the dynamic elements.

The first model assumes the choice of the available power PD, requires a system of justification of the PD type for each branch and the development of methods for determining the PD parameters, which is very difficult. The second model has no problems of implementation according to the steady state algorithm using software tools for calculating the parameters. The random arguments required for this purpose are available active powers and the PDF for load nodes. The voltages in the nodes are distributed according to the normal law, the active available power of the generator nodes (power plants) is distributed according to the binomial law or its continuous approximation. The magnitude of the voltage generating units distributed according to normal law. The functional dependencies in this task are the active and reactive power flows, and the derivatives of their total power and current in the branches [17, 18]. The algorithm for determining the PDs of these FDs is implemented in the same way as obtaining the PDs of mode parameters.

The difference will be the available active power of load nodes in the form of constants. In all the load and generator nodes the voltage value is set as the second parameter, the FD in the form of reactive power flows, as a rule, is practically not used. For each variant of the RV values of the combination process and obtaining the values of each FD embedded in the software tool for calculating the steady-state parameters, it is necessary to calculate additional FD that are of practical interest. Namely: the total power flow in each branch of transformer connections, the current in the circuit or the branch of the linear grid connections. There is a continuous random exchange of electrical quantities along the grid branches, so deterministic methods for selecting the resource capabilities of these connections will be ineffective.

5.1. Actual and available electrical quantities and their imbalances for circuits, connections and grid nodes

The actual electrical values (active, reactive, total power, currents, other values) and their available analogues exceeding the actual ones are used in the power systems operational practice, the electric power industry, as well as educational activities in a deterministic form and give certain practical results, but far from incomplete, because all values are random.

The probabilistic calculations made in the electric power industry widely use the recommendations of statistical studies on the loads [19] (power and voltage) PD approximation by normal PDF and the determination of it parameters. This allows you to set PDF loads of substations and actual power plant loads as RV for the modes calculating problems, electrical quantities in fault condition.

Now, the problem of FD PD obtaining, such as mode parameters and electrical values in case of faults by RV PD using industrial tools is solved almost accurately.

Therefore there is a question of PD use for risks calculation problems excess of the actual electric size over available [20, 21], for example a power flow through a transformer element of it is available power and from decrease in available power plants power of the actual power system power of the transformer element which can be delivered on a grid from power plants. The sum of risks at different actual transformer element has different values. With a minimum total risk value, it's advisable to take the transformer capacity.

Figures 5 and 6 provide a visual representation.

Figure 5 shows the PDFs of the total actual $c_{act}(s)$ and $c_{avl}(s)$ available power plants power through the transformer circuit.

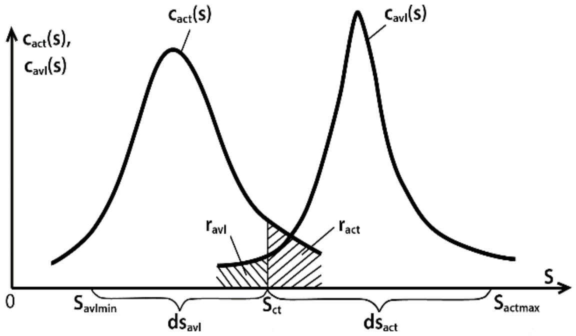

Figure 5. Actual $c_{act}(s)$ and available $c_{avl}(s)$ transformer PDs

where:

$c_{avl}(s)$ – power through the transformer branch of the electrical grid;

S_{ct} – one of the calculated values of the selected available power of the transformer branch;

ds_{act} – the estimated range of the actual power flow exceeding through the transformer branch over the selected available transformer branch power;

S_{actmax} – the maximum value of power through the transformer branch,

ds_{avl} – the calculated range of reduced values of the available power plants power relative to the selected actual transformer power;

S_{avlmin} – the minimum value of the available power plants power, delivered to the desired transformer branch;

r_{act} – risk of the actual power flow exceeding through the transformer branch over the selected available transformer branch power;

r_{avl} – risk of reduced values of the available power plants power relative to the selected actual transformer power.

The total transformer power must be greater than the maximum actual power and less than the minimum available power plants power of the power system flow through the transformer branch.

As shown in Figure 5, the specified power range is practically unlimited, so one of the possible available transformer power values is selected and the risks are found:

- exceeding the actual power through the transformer branch of the selected power S_{ct} (12):

$$r_{act} = p(S > s_{ct}) = \int_{s_{ct}}^{S_{actmax}} c_{act}(s)ds \qquad (12)$$

- reduction of the available power stations power of the selected transformer power S_{ct} (13):

$$r_{avl} = p(S < s_{ct}) = \int_{S_{avlmin}}^{s_{ct}} c_{act}(s)ds \qquad (13)$$

where the S_{avlmin} and S_{actmax} minimum-possible value of the available power flowed from power plants to the transformer and the maximum-possible value of the actual power through the transformer element.

The total risk (14):

$$r_{\Sigma} = r_{avl} + r_{act} \qquad (14)$$

The obtained total risk value corresponds to the available transformer power. However, it is necessary to change S_{ct} several times, then different values of the total risk r_{Σ} will be obtained.

Figure 6 presents the dependence $r_{\Sigma}(S_{ct})$, which shows the minimum risk value r_{Σ} and the corresponding available power S_{opt}, which should be selected as the transformer capacity.

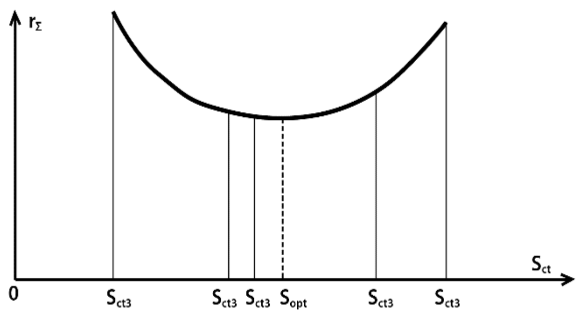

Figure.6. Determination of the total risk at different power

This power is optimal in terms of the transformer overload total risk.

Similarly, the problem of the transmission line optimal cross-section between power plants and substations can be solved. This task can be applied to any transport and grid objects, not only for branch flows, for example, power plants busbars and substations in order to select optimal busbars sections.

6. Conclusion

The developed algorithm and programs for FD PD determining by the RV PD can be applied to frequently used value as imbalance, i.e. differences of the available and actual flows parameters. Imbalance can be positive and negative - essentially random values, but very important for all branches and many nodes.

Knowing the imbalance density, one can determine all the indicators necessary for operation, design and management, and give recommendations on how to change or improve them.

The SIBD method can be important for digital substations, it can not only boost the flexibility and responsiveness of transmission and distribution grids, but also help to make accurate decisions not only in real time, but also in advance, to quickly react to changes grid condition.

The definition of imbalance PD on PD of the actual and available parameter is impractical, because these RVs are dependent on each other and, therefore, in the determining the unregulated averaging of imbalance, unknown conditional PDFs of the available or actual component are required to sum up the joint PDFs values of the available and actual components. Therefore, the FD DP as imbalance should be formed in conjunction with the PD of all the dependent mode parameters and resource values. The unbalance PD formation can also be carried out by algorithms for obtaining the mode parameters density or resource values.

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