Multivariate and Global Particle Swarm Algorithm Optimization in mmWave Massive MIMO for Angle Domain Channel Estimation

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Abstract

Exploitation of massive Multiple-Input Multiple-Output gains for downlink transmission in Millimetre Wave Systems comes at the expense of obtaining accurate channel estimation and computing complexity. Moreover, the fundamental task of massive Multiple-Input Multiple-Output channel estimation is to exploit the characteristics of the channel and sparsity of these multi-antennas systems to simplify complicated spatial structures. We tackle the channel estimation problem in constructing angle domain channel model with multivariate optimization. In the proposed global particle swarm optimization for angle domain aided scheme, the channel estimation design is decoupled into two parts. The simulation results are provided to demonstrate the superior performance of the proposed algorithm over the traditional CS-based channel estimation methods.

Keywords: Millimetre Wave, Multiple-Input Multiple-Output, Global Particle Swarm, Angle Domain Channel

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1. Introduction

With 5G mobile communication networks are built and gradually commercialized, the ability for communication capacity and bandwidth continues to increase.

To meet the tremendous demand for high spectral efficiency, large bandwidth, and high data rate for future wireless communication, e.g., the 5G/6G cellular systems and the Millimetre Wave Systems (mmWave) were considered as promising techniques due to the outstanding advantage of massive available bandwidth in the range from 30GHz to 300GHz [1]-[4].

The mmWave Systems and the massive Multiple-Input Multiple-Output (MIMO) communication system are a key technology for the future mobile communication by exploiting the wireless resources in spatial dimension and using the mmWave band, which has a richer spectrum resources to relieve the pressure of insufficient spectrum resources.

However, the dimensionality of mmWave and massive MIMO channel matrix are large, and it is difficult for a conventional channel estimation algorithms to obtain results of a high accuracy channel estimation in a relatively short time.

Therefore, it is important to guarantee the efficient channel estimation for mmWave massive MIMO systems in low-complexity implementation.

It is recognized the great majority of the benefits of massive MIMO technology in mmWave communication systems, such as high spectral efficiency and high energy efficiency, which is heavily relied on accurate channel state information (CSI) estimation. As one of the main channel estimation methods for mmWave massive MIMO systems, the conventional channel estimation techniques developed for low-frequency MIMO systems are no longer applicable to mmWave massive MIMO systems due to the use of hybrid precoding, large antenna arrays, and the sparsity of mmWave channels [5], [6].

Thus, specific channel estimation techniques, by exploiting high-rank structures for mmWave massive MIMO systems, are proposed [7], [8].

Unfortunately, these common channel estimation algorithms have some disadvantages of high complexity and require a significant overhead.
For this reason, [9]-[12] proposed a compressive sensing (CS) based on a channel estimation strategy that exploits the sparsity of mmWave channels in the angle domain and incorporates a hybrid architecture.

However, the high complexity of the angle domain channel estimation in [9]-[12] is still not effectively addressed due to nonlinear optimization and its effectiveness depends heavily on the restricted isometric property (RIP).

In addition to sparsity, many other structures of mWaves can be used to achieve a channel estimation, such as low-rank structures. A method based on low-rank tensor decomposition is proposed to estimate the channel of a wideband mmWave system [13].

Unlike the CS-based algorithm, the method does not require an oversampled angle grid to discretize the AoAs and AoDs, which has no grid discretization error. [13] showed that the angle expansion of the mmWave channel leads to a low-rank structure that can be used to improve the channel estimation accuracy.

By exploiting the joint sparse and low-rank structure, [14] proposed a two-stage compressed sensing method for mmWave channel estimation that achieves a lower sample complexity compared to conventional CS based on algorithms that only exploit the sparse structure of the mmWave channel. Moreover, several intelligent optimization methods were applied in improving the accuracy and reducing the complexity and worked well.


Due to the sparse characteristic of mmWave, the channel model exhibits sparsity in the angle domain [6], which allows the new estimation strategy to accelerate the convergence of iterations and achieve low complexity.

The fundamental challenge of existing massive MIMO channel estimation is a high computational complexity in intelligent optimization and in complicated spatial structures that bring great difficulties in exploiting the channel characteristics and sparsity of these multi-antennas systems.

In this paper, we propose a novel approach to mmWave massive MIMO channel estimation for angle domain, which we refer to as transformed relaxation optimization into multivariate collaborative optimization, based on the idea that the sparse channel parameters can be integrated for collaborative intelligent search and mutual constraint.

In the proposed global particle swarm optimization for angle domain aided (GPSO-ADA) scheme, the channel estimation design analysed in Section 1 is decoupled into two parts.

In the first part of Section 1, the channel estimation problem is moved from the angular domain relaxation optimization to multi-objective optimization. Based on the sparse property of mmWave channels, the training signal received can be formulated as a high order tensor to estimate the dominated channel parameters, including angle-of-arrivals /angle-of-departures (AoAs/AoDs) and channel gain.

In the second part of Section 1, a global optimal selection PSO is proposed to obtain the near-optimal estimated accuracy. The modifies PSO global search for angle domain channel estimation to shield the interference of the local optimal solution trap and improve computational efficiency. In contrast to SR-IR benchmarks, the performance of the method does not saturate at high SNRs and approximates the Cramer-Rao bound (CRB). Numerical results indicate that there are several advantages of treating the mmWave channel estimation problem in the multivariate-objective and global optimization.

Section 2 develops a system model for the mmWave massive MIMO channel.

Section 3 describes the objective function optimization and the intelligent optimization algorithm for the proposed channel estimation scheme.

Simulation results are given in Section 4.

Section 5 concludes this paper.

2. System Model

In the mmWave massive MIMO system model, the transmitter side is configured with \( N_T \) transmit antennas and \( N_T^{RF} \) transmit RF chains, and the receiver side is configured with \( N_R \) receive antennas and \( N_R^{RF} \) receive RF chains. A phase-shifter-based transmit beamforming structure is used, where the \( N_k \) strip data stream is modulated and mapped to the transmit antennas, and the receive antennas superimpose the information sent by all transmit antennas. The input signal is modulated and mapped to the transmit antenna, and the receive antenna superimposes the information sent by all transmit antennas, then the information is demodulated and decoded and output.

In a practical mmWave massive MIMO system, the number of antennas in a massive MIMO system is much larger than the number of RF chains.

In the training phase, \( N_k \) training symbols are used for channel estimation at the transmitter, and each symbol \( s_k \), \( k = 1, \ldots, N_k \) is transmitted on its own beamforming vector \( f \in \mathbb{C}^{N_T} \), where \( |f_k|^2 = 1 \).

The received signal of the \( k \)-th training symbol \( r_k \) can be expressed as

\[
r_k = H f_k s_k + n_k
\]  

(1)
Assume that the downlink channel is a narrowband and blocks the fading channel, where \( H \) is the \( N_R \times N_T \) downlink mmWave channel matrix.

\( \mathbf{n}_R \) is the noise vector conforming to the complex Gaussian distribution.

At the receiver side, \( N_p \) combinatorial vectors \( \mathbf{w}_p \in \mathbb{C}^{N_R} \), are used to detect the transmitted symbols on each beamforming vector.

We also normalize the combined vectors to \( |\mathbf{w}_p|^2 = 1 \).

Then, the combined signal vector of \( r_k \) can be expressed as:

\[
y_k = W^H s_k + W^H n_k\tag{2}
\]

where \( W = [\mathbf{w}_1, \ldots, \mathbf{w}_{N_p}] \) is the \( N_R \times N_T \) combinatorial matrix used to receive the guide frequency sequence, \( S = \text{diag}(s_1, \ldots, s_{N_p}) \) is the diagonal array of training symbols \( s_k \), \( N = [n_1, \ldots, n_{N_T}] \) is the \( N_R \times N_T \) noise matrix, \( F = [f_1, \ldots, f_{N_T}] \) is the \( N_R \times N_T \) beamforming matrix.

For the channel estimation, the training symbols are assumed to be identical, so \( S = \sqrt{P} I_{N_T} \), where \( P \) is the symbol power and \( I_{N_T} \) is the identification matrix of \( N_T \times N_T \).

Defining \( \mathbf{N} = W^H \mathbf{N} \), the combined received signal can be simplified as:

\[
Y = \sqrt{P} W^H s_k + \mathbf{N} \tag{4}
\]

The channel model of Uniform Linear Arrays (ULAs) for mmWave massive MIMO systems can be described as:

\[
H = \sum_{l=1}^L \alpha_l A_R(p_{l,R}, \theta_{l,R}) A_T^H(p_{l,T}, \theta_{l,T}) \tag{5}
\]

where \( \alpha_l \) denotes the channel gain on \( l \) -th path, \( p_{l,R} \) and \( \theta_{l,R} \) are defined as the angle information on the \( l \) -th path at the receiver.

Similarly, \( p_{l,T} \) and \( \theta_{l,T} \) are the angle information at the transmitting end of the corresponding path.

\( A_R(p_{l,R}, \theta_{l,R}) \) and \( A_T(p_{l,T}, \theta_{l,T}) \) represent the guidance vectors of the receiver and transmitter, respectively. And their matrix sizes are affected by the shape of the antenna array.

\( H \) is influenced by the shape of the antenna array.

For ULAs, \( A_R(p_{l,R}) \) and \( A_T^H(p_{l,T}) \) are determined by only one angle can be described as:

\[
A_R(p_{l,R}) = [1, e^{2\pi i d \sin \frac{\phi_{l,R}}{\lambda}}, \ldots, e^{2\pi i (N_R-1)d \sin \frac{\phi_{l,R}}{\lambda}}]^T \tag{6}
\]

\[
A_T^H(p_{l,T}) = [1, e^{2\pi i d \sin \frac{\phi_{l,T}}{\lambda}}, \ldots, e^{2\pi i (N_T-1)d \sin \frac{\phi_{l,T}}{\lambda}}]^T \tag{7}
\]

Based on (6) and (7), the channel model \( H \) of the mmWave massive MIMO system can be reconstructed as:

\[
H = \sum_{l=1}^L \alpha_l A_R(d \sin \frac{\phi_{l,R}}{\lambda}) A_T^H(d \sin \frac{\phi_{l,T}}{\lambda}) \tag{8}
\]

where \( \phi_{l,R} \triangleq d \sin \frac{\phi_{l,R}}{\lambda} \) and \( \phi_{l,T} \triangleq d \sin \frac{\phi_{l,T}}{\lambda} \) are normalized spatial angles [15].

To further reduce the complexity, (5) can be described as:

\[
H = A_R^\alpha \text{diag}(\alpha) A_T^\alpha \tag{9}
\]

where \( \alpha = [\alpha_1, \alpha_2, \ldots, \alpha_L]^T \) is a sparse matrix, when most of the elements in \( \alpha \) are close to zero.

Only a few non-zero elements are located in different positions of the matrix, whose non-zero elements represent the channel gain on the corresponding path.

Meanwhile, \( \varphi_{R_l} = [\varphi_{R_1}, \varphi_{R_2}, \ldots, \varphi_{R_L}]^T \) and \( \varphi_T = [\varphi_{T_1}, \varphi_{T_2}, \ldots, \varphi_{T_L}]^T \) are the angle information matrices at the receiver and transmitter, respectively.

3. Global Particle Swarm Optimization for Angle Domain aided (GPSO-ADA) Channel Estimation

3.1 Proposed Multi-Optimization Objective Formulation

Taking into consideration the sparse characteristics in (9), the mmWave massive MIMO channel estimation problem can be transformed into a reconfigured zero-parameter optimization problem.

The estimation of \( \alpha \) in (9) is equivalent to obtain the channel path of arrival and departure angles. Thus, the reconstructed channel estimation problem can be described as:

\[
\min_{\hat{\alpha}} \|\hat{\alpha}\|_0 \tag{10}
\]

s.t. \( \|Y - \sqrt{P} W^H H\|_2 \leq \varepsilon \)

where \( \|\hat{\alpha}\|_0 \) represents the number of non-zero elements of \( \alpha \).

\( \varepsilon \) is the fault tolerance parameter associated with the noise statistics.

\( \hat{H} \) is the channel matrix to be estimated.

The estimation results can be obtained by solving the above optimization problem.

The main difficulty in solving (10) is that the zero-parametric problem is computationally unfavourable to find the optimal solution.
The traditional solution is to transform the zero-parity problem into a non-zero-parity problem.

Furthermore, the log-sum function can be used for sparse signal recovery [15]. So, (10) can be reconstructed by replacing the zero parameterization with log-sum function:

$$\min_{\alpha, \phi} Z(\alpha) = \sum_{i=1}^{L} \log(|\alpha_i|^2 + \delta)$$

s.t. \( \| Y - \sqrt{P} \hat{W}_H H F \|_2 \leq \varepsilon \)

\( W = [w_1, \ldots, w_{N_k}], k = 1, 2, \ldots, N_k \)

\( F = [f_1, \ldots, f_{N_k}], k = 1, 2, \ldots, N_k \) \hspace{1cm} (11)

where \( \delta > 0 \) is a transformation parameter to ensure that the logarithmic function is well-defined [16].

By adding a regularization parameter \( \lambda > 0 \) through the Lagrange method, we can further formulate (11) as an unconstrained optimization problem, which yields the multi-optimization function \( G(\alpha, \phi) \).

$$\min_{\alpha, \phi} G(\alpha, \phi) = \left\| Y - \sqrt{P} \hat{W}_H H F \right\|_2$$

(12)

where \( Z(\alpha) = \sum_{i=1}^{L} \log(|\alpha_i|^2 + \delta) \).

(12) can be also described as:

$$\min_{\alpha, \phi} G(\alpha, \phi) = Z(\alpha) + \lambda \left\| Y - \sqrt{P} \hat{W}_H H F \right\|_2$$

(13)

Furthermore, using an iterative surrogate function instead of a log-sum function, the minimization of \( G(\alpha, \phi) \) is equivalent to the minimization of the surrogate function.

$$\min_{\alpha, \phi} S(\alpha, \phi) = \lambda^{-1} \alpha^H D(\alpha) \alpha$$

$$+ \left\| Y - \sqrt{P} \hat{W}_H (A_R(\phi_R))'(\alpha')(A_T^H(\phi_T))' F \right\|_2$$

(14)

where \( (A_R(\phi_R))' \) is the estimated receive angle vector,

\( \text{diag}(\alpha') \) is the estimated channel gain vector

\( (A_T^H(\phi_T))' \) is the estimated transmit angle vector.

Meanwhile, \( D(\alpha) \) can be defined as:

$$D(\alpha) = \text{diag} \left( \frac{1}{|\alpha_1|^2 + \delta} \frac{1}{|\alpha_2|^2 + \delta} \ldots \frac{1}{|\alpha_L|^2 + \delta} \right)$$

(15)

where \( \alpha_i^{(l)} \) is the estimated value of \( \alpha \) at the \( l \)-th iteration.

The minimization of the log-sum function \( G(\alpha, \phi) \) in (13) can be equivalently transformed to the minimization of \( S(\alpha, \phi) \) in (14) [17].

3.2 Global Particle Swarm Optimization for Angle Domain Aided (GPSO-ADA) Channel Estimation

[15] proposed the super-resolution channel estimation based on Iterative reweight (SR-IR) to optimize (14).

However, the problem of SR-IR method is that the optimization is iterative relaxation operation rather than global, which affects the search for the optimal solution of the channel estimation.

PSO is a stochastic optimization scheme based on population optimization techniques, the first proposed by Kennedy and Eberhart. PSO is widely used in optimization problems for its advantages of fewer parameters, easy implementation, and easier leapfrogging of local optimal information. PSO seeks the current optimal solution of the population by pursuing the current optimal solution of the population and continuously adjusting its evolutionary direction to find the global optimal solution [18]. The basic PSO scheme suffers from poor local search ability and insufficient search accuracy [19]. Researches show that the relevant PSO scheme can converge to the global optimal solution with probability 1 [20]. Then, this work modifies PSO global search in [21] for angle domain channel estimation to improve the computational efficiency and optimization. Figure 1 shows the optimization process of PSO algorithm.

![Figure 1. Optimization process of PSO algorithm](image-url)
In Figure 1, the flow of PSO algorithm is introduced in detail, and the influence of various parameters of particle swarm optimization algorithm on the optimization results is explained in detail. It lays a foundation for the improvement of GPSO-ADA algorithm.

The mathematical model of GPSO-ADA multi-optimization objective formulation in (14) can be described as

$$\min f(x_i) = \left| \lambda^{-1}\alpha^i(\alpha + Y - \sqrt{\mathbf{W}^i} \mathbf{H})^2 - f_d \right|, \quad i = 1, \ldots, P$$

$$\mathbf{H} = (A_r(\varphi_R))' \text{diag}^2(a^i) \left( A_T(\varphi_T, i) \right)'$$

s.t. \( |A_r(\varphi_R)| \leq 1, |A_T(\varphi_T, i)| \leq 1 \)

where \( f_d \) is the dynamic search threshold.

The detailed steps of the GPSO-ADA scheme can be divided into:

1. **Initialize particle position and velocity**
   - The reception and emission angles in this optimization scheme satisfy \( |A_r(\varphi_R)| \leq 1, |A_T(\varphi_T, i)| \leq 1 \), the positions \( X = [X_1, X_2, \ldots, X_P] \) and velocities \( v = [v_1, v_2, \ldots, v_P] \) of the randomly generated population particles.

2. **Calculate the fitness function** \( F(X_i) \)
   - From the objective optimization function (16), the fitness function of the improved PSO scheme can be described as
     $$F(X_i) = \lambda^{-1}\alpha^i(\alpha + Y - \sqrt{\mathbf{W}^i} \mathbf{H})^2, \quad i = 1, 2, \ldots, P$$
     $$F^\min_t(X_i) = \min_{i = 1}^{P} F(X_i)$$

3. **Update \( P_{t,\text{best}} \) and \( g_{\text{best}} \)**
   - At \( t \)-th iteration, the fitness value \( F(X_i) \) is calculated for each particle. Meanwhile, \( P_{t,\text{best}} \) is updated if \( F(X_i) \) has a lower fitness value than \( P_{t,\text{best}} \).
     $$P_{t,\text{best}} = X_i, F(X_i) \leq F(P_{t,\text{best}})$$

   - The iterative process always keeps evolving toward the optimal solution. If \( F^\min_t(X_i) \) is smaller than \( g_{\text{best}} \) in each generation of particles, update \( g_{\text{best}} \).
     $$g_{t,\text{best}} = X_i, F^\min_t(X_i) \leq F(g_{\text{best}})$$

4. **Update position \( X \) and velocity \( v \)**
   - The update speed and location play a very important role in PSO scheme that determines if PSO can find the global optimal solution. In this work, a modified PSO based on nonlinear decreasing inertia weight is used.

   The iterative formula of particle position and velocity can be described as:
   $$v_i(t + 1) = \begin{cases} r_1 \cdot \omega(t) \cdot v_i(t) + c_1 \cdot r_1 \cdot \Delta X_1 + c_2 \cdot r_2 \cdot \Delta X_2 \quad \text{if } r_t \cdot v_{\max}(t) \leq \omega_d \cdot v_{\max}(t) \quad \text{otherwise} \\
   \end{cases}$$

   where \( \Delta X_1 = P_{t,\text{best}} - X_i(t), \Delta X_2 = g_{\text{best}} - X_i(t) \), \( t \) is the number of population iterations. \( r_1, r_2, r_t \in [0, 1] \) are random numbers, \( v_{\max} \) is the maximum particle velocity value of generation \( T \), \( T \) is the maximum number of iterations. \( \omega(t) \) is a nonlinear decreasing inertia weight.

   When \( \omega(t) \) is a suitable weight, the modified PSO scheme has a strong global search ability [22].

   Meanwhile, when \( \omega(t) \) is small, the modified PSO scheme tends to accurate local search.

   The expression of nonlinear decreasing inertia weight \( \omega(t) \) can be described as:
   $$\omega(t) = (\omega_{\max} - \omega_{\min} - d_1) \exp \left( \frac{1}{1 + \frac{d_2}{d_3}} \right)$$

where \( \omega_{\max} \) and \( \omega_{\min} \) is the initial and final weight, \( d_1 \) and \( d_2 \) are introduced to change the change rate of the initial weight and the final weight.

The learning factors \( c_1 \) and \( c_2 \) can adaptively adjust the individual learning and population learning of the particles, so that the particle population continuously approaches the individual optimal solution and the global optimal solution. The learning factors \( c_1 \) and \( c_2 \) also reflect the degree to which the particle trajectories are influenced by the individual history information and the population history information.

In an ideal particle state, at the beginning of the search, we should traverse the solution space as much as possible to obtain the diversity of particles.

At the end of the search, the particles should maintain a constant velocity to find the solution in order to shield the interference of the local optimal solution trap.

Therefore, the update formula for the adaptive acceleration coefficient \( c_1 \) and \( c_2 \) is given by:

$$c_1 = c_{11} - c_{12} \cdot \sin \left( \frac{\pi}{2T} \right)$$

$$c_2 = c_{21} - c_{22} \cdot \sin \left( \frac{\pi}{2T} \right)$$

where \( c_{11} \) and \( c_{12} \) represent the initial and final iteration values of \( c_1 \), while \( c_{21} \) and \( c_{22} \) represent the initial and final iteration values of \( c_2 \) respectively.

5. **Maximum speed control**
   - The velocity of the particles \( v_i(t) \) needs to be limited to a maximum velocity range, so as a constraint to control the global search capability.
of PSO, the maximum value $v^{\text{max}}(t)$ needs to be applied in both positive and negative directions.

\[
\begin{align*}
v_{i,j} & \ge v^{\text{max}}, v_{i,j} = v^{\text{max}} \\
v_{i,j} & \le v^{\text{min}}, v_{i,j} = v^{\text{min}}
\end{align*}
\] (24)

(6) Iteration termination condition

This scheme terminates the iteration, if the change of $|F(g_{\text{best}}) - f_{0}|$ in $N$ consecutive generations is within the tolerable $\varepsilon$ range again, or if the particle swarm optimization scheme reaches the termination condition of $T$ generations, and vice versa in the loop (2)-(5).

Scheme 1 describes the flow of the optimization scheme based on GPSO-ADA.

**Scheme 1 Optimization scheme based on GPSO-ADA**

1. **Input:** Noise received signal $Y$, beamforming matrix $F$, combination matrix $W$, iteration termination generation $T$, number of particle populations $P$, and termination threshold $\varepsilon$.
2. **Output:** Estimated angles and path gains of all paths.
3. Randomly initialize the positions $X^0$ and $v^0$ of the initial particle swarm.
4. Setting the parameters of the improved PSO scheme: $w_{\text{max}}, w_{\text{min}}, c_{11}, c_{12}, c_{21}, c_{22}, d_1, d_2, v^{\text{max}}, v^{\text{min}}$.
5. At $t=0$, calculate $F_{\text{min}}(X_i)$ according to equation (17).
6. **While** $t < T$ **do**
7. Update the position $X_i$ and velocity $v_i$ of the particle according to equation (20).
8. Limit the current velocity of the particle according to equation (24).
9. Calculate the fitness value $F(X_i)$ and the contemporary minimum fitness value $F_{\text{min}}(X_i)$ for each particle according to Eq. (17).
10. **If** $F(X_i) \le F(P_{\text{best}})$ **then**
11. $P_{\text{best}} = X_i$
12. **end if**
13. **If** $F_{\text{min}}(X_i) \le F(P_{\text{best}})$ **then**
14. $g_{\text{best}} = X_i$
15. **end if**
16. **If** $|F(P_{\text{best}}) - f_{0}| \le \varepsilon$ **then**
17. cnt++
18. **else**
19. cnt=0
20. **end if**
21. $t = t + 1$
22. **end while**

3.2 Analysis of CRB

It is recognized that the CRB sets a lower bound on covariance matrix of any unbiased estimate of the parameters. It provides a benchmark against which we can compare the accuracy of any unbiased estimator. Moreover, it indicates that the physical impossibility of finding an unbiased estimator with variance less than the bound [23].

Meanwhile, some studies have been reported on the Cramer-Rao bounds (CRBs) for angle domain channel estimation [24]-[30].

Define $\xi = [\alpha, \varphi]^T$ as the vector to be estimated and intelligently searched. The CRB matrix can be defined as the inverse of the Fisher information matrix (FIM) $J$:

\[
\text{CRB} \ge J^{-1}
\] (25)

If the joint conditional probability density function (PDF) is $G$, the FIM is determined by [28]:

\[
J = -E \begin{bmatrix}
\frac{\partial^2 \ln G}{\partial \xi_1^2} & \frac{\partial^2 \ln G}{\partial \xi_1 \partial \xi_2} \\
\frac{\partial^2 \ln G}{\partial \xi_2 \partial \xi_1} & \frac{\partial^2 \ln G}{\partial \xi_2^2}
\end{bmatrix}
\] (26)

Let an array of $m$ sensors receive signals the from $N_T$ ($L < N_T$) narrowband far-field sources with unknown angle information.

The $N_T \times 1$ array vector can be modelled as:

\[
y(t) = aA(\varphi)s(t) + n(t), t = 1, \ldots, N
\] (27)

where $y(t) = [y_1(t), \ldots, y_{N_T}(t)]^T$ is the $N_T \times 1$ array vector,

\[
A(\varphi) = \begin{bmatrix}
1, e^{2\pi j \varphi}, \ldots, e^{2\pi j(N_T-1)\varphi}
\end{bmatrix}^T,
\]

$s(t) = [s_1(t), \ldots, s_{N_T}(t)]^T$, $n(t) = [n_1(t), \ldots, n_{N_T}(t)]^T$ is a complex sample function from an analytic white Gaussian noise process with (mono-lateral) power spectral density (PSD) $2N_0$.

The joint PDF of $G$ can be described as [29]:

\[
G = \exp \left\{ -\frac{1}{4N_0} \int_{T_0} \|y(t) - E_n(y(t))\|^2 dt \right\}
\] (28)

where the subscript $n$ in (28) indicates that the statistical expectation $E$ is to be computed with respect to the noise vector $n(t)$, which is the stochastic component of the estimation model.

We assume to observe $y(t)$ for a time lag $T_0$ equal to one symbol time $T_s$ only [30].

From (27) and (28), we obtain:

\[
\frac{\partial^2 \ln G}{\partial \xi_1^2} = \frac{N_T}{N_0/P_sT_s}
\]

\[
\frac{\partial^2 \ln G}{\partial \xi_2^2} = \frac{\partial^2 \ln G}{\partial \xi_2 \partial \xi_1} = 0
\]

\[
\frac{\partial^2 \ln G}{\partial \xi_2 \partial \xi_1} = \frac{(2\pi \varphi)^2 N_s(N_T-1)(2N_T-1)}{6N_0/P_sT_s}
\]

(29)

where $P_s$ denotes the transmit power.

So, the CRB is found to be:

\[
\text{CRB}(\alpha) = \frac{N_0/P_sT_s}{N_T}
\]

\[
\text{CRB}(\varphi) = \frac{24}{(2\pi \varphi)^2N_s(N_T-1)(2N_T-1)N_0/P_sT_s}
\]

(30)

After obtaining the CRB for angle domain estimation, an upper limit of estimation accuracy can be obtained. If the resource allocation quantification and the correlation function fitting are performed through the upper limit of accuracy, a better energy efficiency optimization can be achieved.

4. Simulation Results

The numerical results are provided to verify the effectiveness of the proposed GPSO-ADA channel estimation (see Table 1).
Table 1. Basic parameters of GPSO-ADA

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of transmit antennas $N_T$</td>
<td>4-128</td>
</tr>
<tr>
<td>Number of receive antennas $N_R$</td>
<td>4-128</td>
</tr>
<tr>
<td>The number of pilot sequences $N_k$</td>
<td>36</td>
</tr>
<tr>
<td>Transmitter radio frequency (RF) chains number $N_{TF}$</td>
<td>4</td>
</tr>
<tr>
<td>Receiver radio frequency (RF) chains number $N_{RF}$</td>
<td>4</td>
</tr>
<tr>
<td>Number of propagation paths $L$</td>
<td>1-3</td>
</tr>
<tr>
<td>Wavelength $\lambda$</td>
<td>0.01m</td>
</tr>
<tr>
<td>Antenna interval $d$</td>
<td>0.005m</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>28GHz</td>
</tr>
<tr>
<td>System geometry</td>
<td>ULAs</td>
</tr>
</tbody>
</table>

We consider the ULA’s geometry, so that the auxiliary beam pair-based channel estimation [31], and the SR-IR channel estimation can be adopted for performance comparison [15].

The normalized mean square error (NMSE) is used to measure the channel estimation accuracy, which can be described as:

$$\text{NMSE} = \frac{\mathbb{E} \left[ \sum\limits_{i=1}^{L} |H(i) - \hat{H}(i)|^2 \right]}{\mathbb{E} \left[ \sum\limits_{i=1}^{L} |H(i)|^2 \right]} \quad (31)$$

In our simulations, we consider that the BS is equipped with a ULA with $L = 3$, $d = \lambda/2$, $N_R = N_T = 128$, $N_{RF} = N_{RF}^T = 4$.

An angle-domain Rician fading channel model is assumed, to model the channel with a none-line-of-sight (NLoS) component.

Figures 2 and 3 compare the NMSE performance with different channel estimation schemes against the SNR, under none-line-of-sight (NLoS) and line-of-sight (LoS) channels.

For comparison, the CRB of the angle-domain estimation is presented as the benchmark. As the SNR increase, the proposed GPSO-ADA channel estimation is approximate to the theoretical CRB.

It also shows that the proposed channel estimation scheme outperforms the other channel estimation schemes even with a smaller pilot overhead.

When NMSE $= 10^{-2}$, the SNRs of GPSO-ADA is about 6dB, which is obviously better than SR-IR method, because our methods utilize not only the multivariate optimum design but also the structural feature of global PSO. The modified PSO increases the ability to break away from the local optimum.

The simulation for GPSO-ADA shows that the modified PSO algorithm can obtain better efficiency and global optimization.

As can be readily observed, the numerical results match exactly with the analytical results, thereby validating the correctness of the analytical expressions.

Figure 4 compares the NMSE performance of the proposed channel estimation algorithm under various $N_T$ and $N_R$. 

Figure 3. NMSE performance comparison of different channel estimation schemes under NLoS channel
In this simulation, \( N_R^{RF} = N_T^{RF} = 4 \) and SNR=20dB are adopted. The comparison of the curves indicates that our proposed methods, based on GPSO-ADA out, perform the existing methods based on SR-IR channel estimation, because the CSI can be obtained directly in the angle domain and the sparsity features are fully leveraged with the aids of the excellent global optimization ability of the PSO.

Moreover, as expected in CRB, increasing the number of transceiver antennas can greatly reduce the NMSE. This is because with a large number of \( N_T \) and \( N_R \), more angle-related data can be obtained based on which we can estimate the angles more accurately. Additionally, the significant enhancement of the NMSE performance of the channel estimation in angle domain is not required for high computational complexity in our proposed GPSO-ADA based scheme.

5. Conclusions
In this paper, we derived the GPSO-ADA channel estimation scheme for the joint optimization of AoA, AoD and multi-path gains for mmWave massive MIMO systems. We investigated the local optimal solution trap of mismatch relaxation iteration and the performance loss problem in all existing iterative reweight-based channel estimation.

To tackle this challenge, the channel estimation problem was moved from the angular domain to the multi-objective optimization. Then, the modified global PSO was proposed to obtain the near-optimal estimated accuracy on top of this model.

To evaluate the benefits of the proposed scheme, the CRB of angle domain channel estimation was presented as the benchmark. It is shown that the proposed GPSO-ADA channel estimation scheme is approximate to the theoretical CRB, and it outperforms the other channel estimation schemes.

6. Bibliographic References


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